

Motif Learning Facilitates Sequence Memorization and Generalization

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ABSTRACT

Whether it is listening to a piece of music, learning a new language, or solving a mathematical equation, people often acquire abstract notions in the sense of motifs and variables — manifested in music, grammatical categories, or mathematical symbols. How do we create abstract representations of sequences? Are these abstract representations useful for memory recall? In addition to learning transition probabilities, chunking, and tracking ordinal positions, we propose that humans also use abstractions to arrive at efficient sequence representations. We propose and study two abstraction categories: projectional motifs and variable motifs. Projectional motifs find a common theme underlying distinct sequence instances. Variable motifs define symbols manifested in varying instances. We show that both motif categories help a model to reduce sequence representation complexity via encoding sequences in an abstract space, thereby facilitating the model to learn more efficiently and transfer to novel sequences. In two sequence recall experiments, we train subjects to remember sequences with projectional and variable motifs, respectively, and examine whether motif training benefits the recall of unseen novel sequences sharing the same motif. Our result suggests that training variables and projectional motifs improve recall accuracy, specifically on transfer lists but not randomly created control lists relative to independent control groups. Our study suggests that humans construct efficient sequential memory representations according to the two types of abstraction we propose, and it shows that creating these abstractions benefits learning and out-of-distribution transfer. Our study paves the way for a deeper understanding of human abstraction learning and generalization.

Introduction

When the iconic notes strike: GGGE♭, FFFD, — Beethoven's Fifth Symphony comes immediately to our mind. As the music progresses, we note the change of motif to GGGB or GGGC, variations in forms and voices, one at each step. Our ability to effortlessly identify those forms of abstract motifs endows us with an ability to learn mathematics, languages, and various forms of art. From representing "x" as a variable to perceiving 'noun' as a category including "cats", "dogs", and "elephants", these abstract motifs automatically come to our mind and help us to memorize sequences and generalize to novel situations. How do we abstract motifs from perceiving sequences? What advantages does this ability confer in terms of memory representations and transfer? More importantly, how do we construct an abstract representation during learning?

Previous studies on sequence learning primarily focused on artificial grammar learning¹ and grammatical judgment tasks². Usually, a fixed transition matrix between syllables generates grammatically valid sequences³. After exposure to these grammatically valid sequences, subjects can distinguish grammatical and ungrammatical sequences at test¹. Further research has been conducted to determine whether sequence learning extends beyond the learning of first-order transition probabilities. An alternative proposal is chunking: upon practice, humans can learn and parse sequence by disjunctive sequential chunks. Models including PARSER⁴, the hierarchical chunking model⁵, and TRACX^{6,7} capture producing segmentations of sequences by extracting repeated recurring units from a continuous input stream. Yet few studies have looked at the acquisition of abstract patterns in sequences: Marcus et al. conducted a study to test infants' ability to learn "algebraic structure" in sequences. The study involved exposing 7-month-old infants to sequences such as AAB and CCD. After exposure, the infants were more likely to direct their gaze toward novel sequences sharing the same structure, such as DDF, rather than toward a different structure, such as KTK. Infants' ability to capture 'abstract algebraic structure'³ in sequences cannot be explained by learning transition probabilities or chunks. The algebraic structure was defined as a mapping of multiple items onto a single sequential item.

In this work, we zoom in, refine, and categorize different forms of abstract sequential structures. We define and differentiate between two algebraic abstractions: "projectional motifs", which are patterns derived from sequences using a projectional function, and "variable motifs," which include patterns that involve both concrete and variable elements. Furthermore, we propose a model that learns abstract representations incorporating components from transition probabilities, chunks, and motifs to reduce memory complexity. We look at the learning and transfer abilities of subjects in two sequence recall experiments.

34 Our results suggest that humans learn both types of motifs for transfer. Whereas associative learning and chunking are both
 35 necessary to account for sequence recall performance during training and transfer, our proposed mechanisms of abstraction
 36 explain an additional substantial portion of the systematic variance in people's sequence recall accuracy.

37 **A Taxonomy of Sequence Motifs**

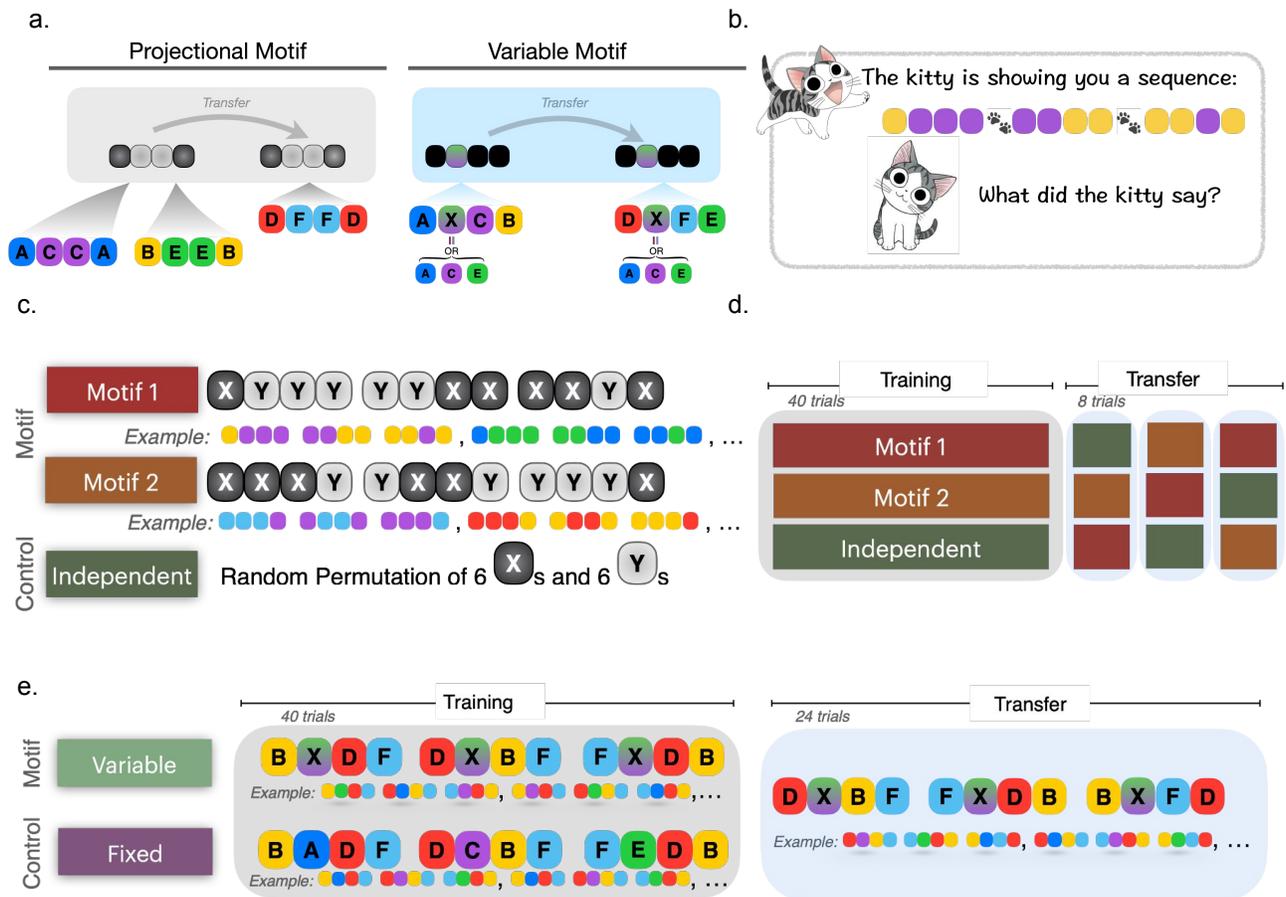


Figure 1. Taxonomy of motifs and experimental design a. A taxonomy of sequence motifs over what is classically known as algebraic structures in sequences. Projectional motifs refer to patterns of sequences in a projected space that are mapped from the concrete sequence space by a projection function. In the example being shown, the projection function finds the distinct items in the sequence and maps sequential observation into a binary sequence in the projectional space. Variable motifs refer to identifying multiple sequential components as a variable. Such a variable is identified when any of the sequential components it entails is identified. We hypothesized that participants could learn both types of motifs through practice and exploit their knowledge of both motif types in memorizing and generalizing to novel sequences. b. We study motif learning in a sequence recall task. Participants are instructed to remember a sequence of 12 colors displayed one after another in three groups of four items separated by a pair of paws after each group. c. Experiment 1 studies learning projectional motifs. Participants are divided into three groups. Two motif groups (Motif 1 and Motif 2) and one control group (Independent). Each group is first trained on their respective motif or random sequences (Independent) and then tested on randomly interleaved transfer blocks of three sequence types. There are no overlapping sequences between all transfer blocks and training blocks. d. Experiment 2 studies learning variable motifs. The variable group is trained on sequences with an underlying variable motif. That is, the second position of each subsequence display is randomly drawn among three colors (purple, blue, or green). The fixed group is trained to recall fixed sequences. Both groups are then subsequently tested on novel sequences sharing the variable motif.

38 We define sequence motifs as underlying sequence patterns that are not on the item level but only detectable after performing
 39 transformations on sequences of items. We define and study two types of sequence motifs: projectional and variable. An

40 illustration of the two motif types is shown in Figure 2.

41 **Projectional motif** refers to applying a transformation function onto sequences that maps the sequential items to a lower-
42 dimensional projected space. The common motif shared between GGGE \triangleright , and FFFD that begins Beethoven’s Fifth Symphony
43 is one example. In our experiment, this will correspond to seeing a sequence of consecutively displayed colors in one trial such
44 as yellow, purple, purple ..., and in another trial, seeing blue, green, green, ..., as shown in Figure 1.

45 The second type of sequence motif is called **variable motif**. It refers to identifying a variable entity as a part of the sequence.
46 A variable entity can entail several mutually exclusive concrete sequence instances. The variable entity is perceived when any
47 of its entailing sequence instances is perceived. Once identified, the variable identity is treated as a sequence observational
48 unit and can be remembered in conjunction with the invariant part of the sequence. In Beethoven’s Fifth, the progression from
49 GGGE \triangleright to GGGB and later GGGC underlies one variable motif. In our experiment, as Figure 2 illustrates, the variable "X",
50 described by a gradient-colored box, is identified if any of the blue, purple, or green box is identified. The definition of variable
51 motifs bears resemblances to identifying the grammatical category structures in language, such as the concept of a noun entails
52 a set of possible noun words. Or the process of isolating a variable x when working with algebraic expressions such as $x + 3$.
53 While 3 represents a concrete number, x becomes a variable that represents the entity of "anything".

54 Here, we construct a model that learns abstraction in the case of projectional and variable motifs to reduce representation
55 complexity. The model first tracks the transition probabilities in an abstract space and then gradually chunks sequential elements
56 together. We will test the predictions of our model in two experiments.

57 Results

58 We study the effect of memorizing projectional and variable motifs in sequences by asking the following questions: 1. Are
59 sequences constructed according to an underlying motif memorized more accurately than randomly generated sequences, and 2.
60 Are novel sequences, which consist of not seen items and share the same motif as sequences participants learned previously,
61 recalled more accurately than random sequences? We ask these questions in two experiments, each studying one proposed
62 motif type. Furthermore, we hypothesized that memory representations become less complex when a motif is learned. We
63 implemented this assumption in our computational model that continuously finds recurring motifs in sequences.

64 In Experiment 1, we tested whether people can learn and transfer sequences described by a projectional motif as shown in
65 Figure 2. In Experiment 2, we tested whether subjects remember novel sequences better when these sequences share the same
66 variable structure as shown in Figure 2.

67 Taken together, we implemented learning structured motifs as a memory compression strategy in a computational model.
68 The model exhibits similar learning and transfer behavior to participants in two sequence recall experiments testing each motif
69 type.

70 Motif learning model

71 We put forward a model that learns to memorize long sequences via a combination of three strategies: associative learning,
72 chunking, and motif abstraction.

73 **Associative Learning** When a sequence is presented to the model, the model keeps track of the observational frequencies
74 and the transition frequencies between subsequently presented items. Once an item has been identified, its occurrence frequency
75 will increment by 1, and so will the transition frequency between the current item and the previously identified item. Meanwhile,
76 all frequencies are subject to memory decay via multiplying the count of both the marginal and transition frequency entries by a
77 decay parameter $\theta \leq 1$.

78 **Chunking** Apart from associative learning, the model also remembers sequences by chunking. This part of the model is
79 based on the hierarchical chunking model described in our earlier work (HCM⁸). The model stores learned chunks in long-term
80 memory. These chunks are used in addition to observation and transition frequencies to parse the instruction sequence. The
81 model keeps track of the marginal frequencies of chunks and the transitional frequencies between chunks. A new chunk is
82 created by combining two correlated consecutively occurring chunks into a longer chunk. The combined chunk is then added to
83 the memory of the model. This feature enables the model to learn longer and longer sequences with practice. A picture of how
84 memory chunks are acquired during learning is: at the beginning of the training block, the model stores no sequence segments,
85 and therefore, the model parses the first instruction sequence as 12 sequences of unitary length. These unitary sequential
86 chunks are stored in memory as distinct units. As the model learns to combine previously learned chunks into larger chunks,
87 these larger chunks are, in turn, used to parse the upcoming instruction sequences. During the parsing process, the memory
88 chunk of the largest size, consistent with the upcoming instruction sequence, is identified. In this way, the longer the sequence
89 segments the model has learned, the fewer segments are needed to parse the instruction sequence, and the further the model can
90 predict the sequence. In this way, the model builds up a stable memory representation of sequences over practice by combining
91 pre-existing stable representation of memory sequences in long-term memory^{9,10}.

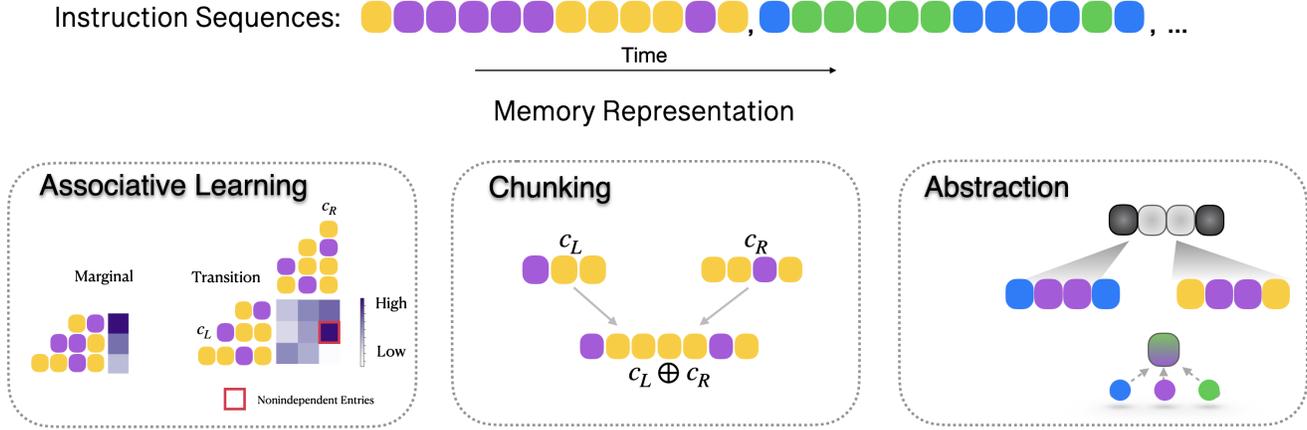


Figure 2. Motif learning model. Upon observations of instruction sequences, the model acquires the transition frequencies between the learned chunks, combines previously learned chunks into new ones, and looks for abstract representations to compress its sequence memory.

92 We also formalize memory chunks based on their occurrence probabilities, consistent with memory models with memory
 93 strength increasing with practice⁶. The lower bound on the number of bits needed to encode this chunk c to be distinguished
 94 from other chunks in memory is $-\log_2 P(c)$. The more probable that a chunk occurs in the instruction sequence, the less the
 95 memory encoding cost.

96 **Abstraction** When the same sequence is presented repeatedly, subparts of the sequence will gradually combine through
 97 the chunking process. However, this process is slow because it requires several repetitions of the chunk. This is especially
 98 problematic when the instruction sequence repeats only rarely, since each unique sequence has only a small probability in the
 99 sequence observation space, and the number of repetitions would have to be increased for the chunking process to build up a
 100 memory of the whole sequence. We propose the learning of projectional and variable motifs as two mechanisms to reduce the
 101 complexity of memory representations.

102 **Abstraction via learning projectional motifs** The model identifies two unique items to describe the sequence and assigns
 103 X to the first occurring item and Y to the second item. In this case, X and Y represent separate entities in the projectional
 104 motif space. This will be one way that the sequence can be transformed into a lower-dimensional space, in which only two
 105 dimensions exist.

106 Once observational sequences are projected onto a lower dimensional projectional motif space, the model learns the
 107 sequence via associative learning and chunking and builds up memory representations of sequences by combining correlated
 108 consecutively occurring chunks in the projectional motif space.

109 For example, upon seeing ACCC, BDDD, and FEEE sequences, the model will map all three sequences onto the same
 110 sequence in the projectional motif space: XYYY. Originally, there needed to be six dimensions to describe the observational
 111 sequence, each representing the binary indicator of observing each letter. The abstraction process enables all three sequences
 112 to be described by the same pattern in an abstract projectional space with two dimensions. Without abstraction, if each of
 113 the three sequences occurs uniformly likely, then the minimal encoding length to distinguish between the three subsequences
 114 shall be $-\log P(\frac{1}{3})$. But once the projectional motif has been identified, it explains all observational sequences and demands
 115 significantly less encoding memory of $-\log P(1)$.

116 **Abstraction via learning variable motifs** Under the demand of learning to remember long sequences, an alternative way
 117 to compress sequence representation is to learn variables. A variable is an abstract sequence entity that entails a set of concrete
 118 sequence entities/chunks. The model identifies the variable identity whenever any of its entailing entities is identified.

119 The abstraction model discerns variables by analyzing the structure of the transition matrix. Specifically, the model identifies
 120 structural patterns within a series of sequential observation chunks that share a common precursor and successor. For instance,
 121 if the model observes that entity A transitions to B , C , and E , and further notes that B , C , and E each transition to F (as reflected
 122 in the transition matrix), it will recognize a new variable encompassing B , C , and E . This variable becomes identifiable when
 123 any of the elements B , C , or E are detected.

124 Once a variable entity has been learned, it is parsed and identified as one entity to join forces with associative learning
 125 and chunking. In this way, the variable helps the learning agent discover an overarching pattern in the sequence, which would
 126 otherwise demand more sequence observations to be learned as separate memory chunks.

127 The mechanism of variables naturally leads to sequence compression. For example, assume the following subsequences:
128 BADF, BBDF, and BCDF have been observed to occur equally likely; each subsequence demands a minimal encoding
129 complexity of $-\log P(1/3)$. As soon as a variable X is identified to entail A, E, or C, then the chunk BXDF would suffice to
130 explain all three observational instances, and this chunk demands a minimal encoding length of $-\log P(1)$.

131 The model learns memory pieces by combining chunking and associative learning. On top of that, sequence abstraction
132 processes, including projectional transformation and identifying variables, help the model to locate recurring motifs in the
133 abstract space, capable of explaining a larger number of sequence observations and thereby learning faster and compressing
134 further.

135 A natural benefit of learning abstract motifs is generalization to novel, unseen sequences sharing the same motif structure.
136 The previously learned projectional or variable motifs can be reused to remember novel sequences, facilitating novel sequence
137 acquisition and compression.

138 The model predicts that subjects looking for the minimal complexity representation to learn sequences should behave in the
139 following ways:

- 140 • When there are underlying projectional or variable motifs in the sequence, subjects' representation of the sequence shall
141 decrease in complexity when more sequences are presented with the same motif type.
- 142 • Subjects who benefit from learning motifs from training sequences will exploit their previously learned motif structure.
- 143 • In the case of projectional motif, motif structure that has been learned before will be exploited to memorize a novel
144 sequence that has never been observed/seen by participants.
- 145 • When subjects learn the representation of a variable and extrapolate it as a sequential unit to be combined with the
146 unvarying part of the sequence, the variable as a concept will be reused when novel sequences sharing the same variable
147 but distinct varying sequence structure need to be remembered.

148 We will test these predictions in detail in the following two experiments.

149 **Experiment 1: Projectional motifs**

150 Experiment 1 tested how projectional motifs could help memorization and transfer by instructing participants to memorize long
151 sequences. In a sequence recall task, participants were instructed to play a memory game and to memorize 12 consecutively
152 displayed colors by a cartoon cat. After the instruction, they had to recall the sequence by pressing the keys corresponding to
153 the colors.

154 Unbeknownst to the participants, the instruction sequences contained underlying motifs. As shown in Figure 2, the motifs
155 consisted of two distinct variables, X and Y, and individual motifs were constructed by arranging patterns of Xs and Ys. All
156 sequences contained an equal amount of 6 Xs and 6 Ys to control for stimulus-specific habituation effects. Each participant was
157 randomly assigned to one of the two motif groups (Motif 1; Motif 2), or to a control group (Independent). Motif 1 followed the
158 pattern XYYY YYXX XXYY, while Motif 2 adhered to the format XXXY YXXY YYXX. In the motif groups, the underlying
159 motif remained consistent across trials. Conversely, in the Independent group, a permutation of 6 Xs and 6 Ys was generated
160 for each trial. The instruction sequences were finalized by mapping X and Y to two distinct colors.

161 The task was divided into training and transfer blocks. The training block comprised 40 trials, after which participants
162 proceeded to three randomly ordered transfer blocks, each testing for Motif 1, Motif 2, and the Independent sequences with 8
163 trials. To ensure that no sequences in transfer blocks appeared in the training block, the six colors were divided into two sets:
164 the training set with four colors and the transfer set with the remaining two colors. Participants were not informed about block
165 transitions.

166 **Model Prediction**

167 **Reducing representation complexity through Projectional Motifs** In the case of projectional motifs, a rational agent that
168 looks for minimal complexity representations shall acquire the unchanging motifs during learning since motifs in the abstract
169 projectional space explain more instances of sequences compared to memorizing concrete sequence instances.

170 Our hypothesis posits that an underlying motif within training sequences in a projectional space will enhance memory and
171 out-of-distribution transfer. In this context, a sequence of length n can be conceptualized as a point within an n-dimensional
172 space, and out-of-distribution refers to the capacity to transfer the representation to sequences never encountered during training.
173 We anticipate improved learning and memorization performance during training for both motif groups and positive transfer
174 when the two groups are tested on motifs of the same type.

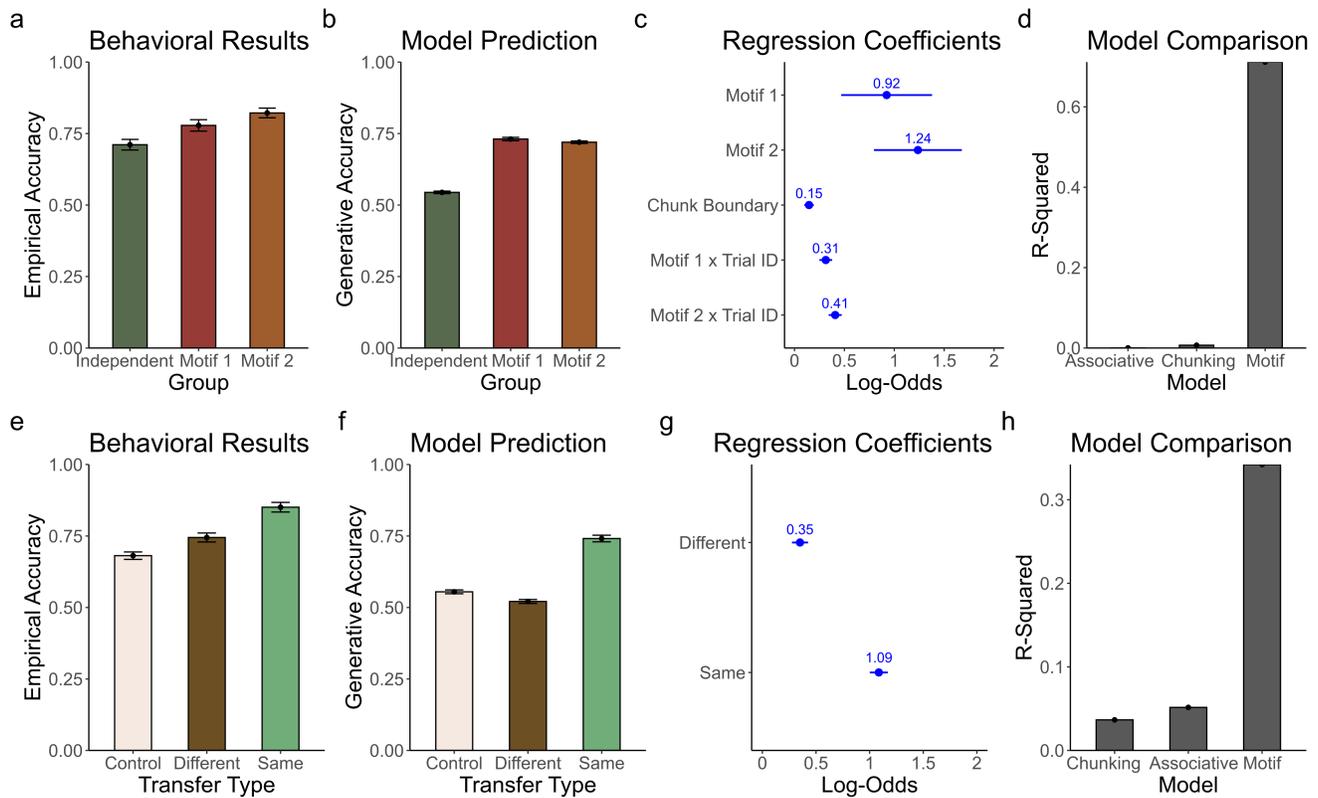


Figure 3. Model simulation and behavioral results for learning and transferring projectional motifs. a. Recall accuracy is higher during the training block in the motif groups than in the control group. b. Model prediction of sequence recall accuracy training on participants' instruction data. c. Regression coefficients of the linear mixed-effect model predicting recall accuracy during the training block. d. Generative accuracy as simulated by the motif learning model correlates with the empirically observed sequence recall accuracy across groups during the training trials. e. Behavioral results of group-wise recall accuracy across three categories of transfer. Same: Motif 1 – Motif 1 and Motif 2 – Motif 2; different: Motif 1 – Motif 2, and Motif 2 – Motif 1; control: Independent – Motif 1, and Independent – Motif 2. f. Simulation transfer results. g. Beta coefficients of the logistic regression predicting recall accuracy during the transfer blocks. h. Correlation between the simulated recall accuracy and participants' recall accuracy

175 **Training**

176 **Behavioral Results** We first compared sequence recall accuracy amongst the three groups in the training block as shown in
177 Figure 3 a. We fitted a linear mixed-effects regression model onto participants' trial-wise sequence recall accuracy, assuming
178 a random intercept over participants and excluded trials that were immediate repetitions. We observed a significant effect
179 of group ($\chi^2(2) = 31.77, p \leq 0.001$), suggesting that participants in the Motif 1 group ($\hat{\beta} = 0.12, se = 0.03, t(103) = 4.28,$
180 $p \leq 0.001$) and the Motif 2 group ($\hat{\beta} = 0.16, se = 0.02, t(103) = 5.91, p \leq 0.001$) recalled sequences more accurately during
181 the training blocks than those in the Independent group; this result is consistent with our prediction that participants would
182 remember sequences containing motifs better.

183 **Model Simulation** We compared the behavioral results with the model predictions. We used the same sequences instructed
184 to the participants to train the motif learning model, which creates memory representations of sequence motifs from the
185 observational sequences in an abstract space. We then generated sequences based on the representations learned by the model
186 up to the current time point. We came up with generative accuracy as a surrogate for sequence recall accuracy. The generative
187 accuracy was the edit distance between a generative sequence sampled from the model and the instruction sequence in a
188 particular trial. Figure 3 b shows the average generative accuracy of the model. We observed a significant effect of group
189 ($\chi^2(2) = 44.77, p \leq 0.001$), suggesting that participants in the Motif 1 group ($\hat{\beta} = 0.10, se = 0.01, t(1477) = 7.94, p \leq 0.001$)
190 and the Motif 2 group ($\hat{\beta} = 0.07, se = 0.01, t(1477) = 5.28, p \leq 0.001$) recalled sequences more accurately during the training
191 blocks than the independent group. Similar to participants, the model remembered sequences with underlying motifs more
192 accurately.

193 **Regression Coefficient** Apart from having higher average recall accuracy, both motif groups improved their recall accuracy
194 faster. As shown in Figure 3 c, we analyzed participants' recall key-press correctness by fitting a logistic regression model
195 assuming a random intercept of each participant and a random slope over individual serial positions (explanation on random
196 effect structure selection in method section). We observed an effect for both Motifs (for Motif 1: $\beta = 0.92, se = 0.23, z = 3.97,$
197 $p \leq 0.001$; for Motif 2: $\beta = 1.24, se = 0.22, z = 5.50, p \leq 0.001$). Apart from that, we observed an interaction effect between
198 the trial number and group ($\chi^2(2) = 51.69, p < 0.001$). Participants in the Motif 1 group improved their recall accuracy at a
199 faster rate than participants in the Independent group ($\beta = 0.31, se = 0.03, z = 9.64, p \leq 0.001$); the same effect was present
200 for the Motif 2 group ($\beta = 0.41, se = 0.03, z = 12.23, p \leq 0.001$). Thus, people improved faster on remembering sequences
201 with fixed motifs than sequences without.

202 **Model Comparison** We compared the recall accuracy of the motif learning model with two alternative models: an
203 associative learning model and a chunking model. The motif learning model constructs memory pieces by combining chunking,
204 associative learning, and abstraction via learning projectional motifs. The chunking model contains the same components
205 except for abstraction. The associative learning model learns the first-order transition between observed sequential items. We
206 gave the same instruction sequence to all three models and thereby arrived at an average recall accuracy for each model on each
207 proceeding experimental trial.

208 We then regressed the generative accuracy of each model onto empirical accuracy and evaluated the goodness of fit by
209 computing the R-squared value. The R-squared measure determines the proportion of variance in the behavioral results that
210 the model prediction can explain and shows how well the data fit the regression model. As shown in Figure 3 d, the motif
211 learning model ($R^2 = 0.71$) explained more variance in the behavioral result than a chunking model ($R^2 = 0.006$) that did not
212 abstract. This suggests that abstracting the sequence via projecting the sequence onto the motif space is a critical component
213 that captures human behavior in this task.

214 Comparing the motif learning model to an associative learning model shows that abstraction alone isn't enough to explain
215 the results. The associative learning model factors in marginal and transition probabilities in the sequences but doesn't learn
216 chunks. Additionally, it explains very little of the variance in human behavior, with $R^2 = 0.0002$, compared to the motif
217 learning model. This result suggests only learning the association between items in the projected motif space is insufficient;
218 combining the previously memorized memory chunks together into longer memory chunks is also vital to explaining human
219 learning progress.

220 **Transfer**

221 We then assessed whether training on motifs affected participants' ability to memorize novel sequences in the transfer blocks.

222 **Behavioral Results** We compared participants' performance in the transfer blocks grouped by three transfer types relative
223 to the training block types: Same (Motif 1 - Motif 1, Motif 2 - Motif 2), Different (Motif 1 - Motif 2, Motif 2 - Motif
224 1), and Control (Independent - Motif 1, Independent - Motif 2). Shown in Figure 3 e, we observed a significant effect of
225 transfer type ($\chi^2(2) = 94.66, p \leq 0.001$) on recall accuracy. Participants remembered novel sequences with the same motifs
226 more accurately compared to control ($\hat{\beta} = 0.17, se = 0.01, t(154) = 11.22, p < 0.001$). Surprisingly, we also observed that
227 participants benefited from transferring to a different motif type compared to control ($\hat{\beta} = 0.06, se = 0.01, t(154) = 4.21,$
228 $p \leq 0.001$). Consistent with our hypothesis, training on sequences with motifs helps participants learn novel sequences sharing

229 the same motifs.

230 **Model Prediction** Similarly, we evaluated the recall accuracy of the motif learning model on the transfer blocks. Figure 3 f
231 shows the generative accuracy of the motif learning model grouped by transfer types. Similar to participants, the model recalled
232 novel sequences with motifs better after it had been trained on the same motif ($\chi^2(2) = 68.25, p \leq 0.001$), compared to having
233 been trained on neither motif ($\beta = 0.14, se = 0.02, t = 7.82, p \leq 0.001$). Different from the participant: It is harder for the
234 model to transfer to an alternative motif type ($\beta = -0.04, se = 0.02, t = -2.38, p = 0.02$) than the control. We inspect this
235 discrepancy further in the discussion section.

236 **Regression Coefficients** We looked at participants' correctness of recall key presses by fitting a logistic regression model,
237 assuming a random intercept of participants and random slope over individual serial positions and trial numbers (Figure 3 g).
238 We found that the transfer types affect the recall key press correctness ($\chi^2(2) = 679.46, p \leq 0.001$). Participants who have
239 been tested on the same motif as they had been trained on (m1 - m1 and m2 - m2) ($\beta = 1.09, \sigma = 0.04, z = 25.28, p \leq 0.001$)
240 are more likely to recall the correct item compared to control. This result resonates with our linear mixed-effect analysis on
241 recall accuracy. Interestingly, participants who were tested on a motif different from their training motif also did better than the
242 control ($\beta = 0.35, \sigma = 0.04, z = 9.12, p \leq 0.001$). We discuss the implications of this finding further in the discussion section.
243 Additional regression coefficients that confirm practice effect, recency effect, and chunk boundary effect are reported in the
244 supplementary information.

245 **Model Comparison** We then compared the resemblance to human behavior between the motif learning model, the
246 associative learning model and the chunking model (Figure 3 h) during the transfer blocks. Since all three models change their
247 representation when the training schedule switches from training to the transfer blocks, we can compare the generative accuracy
248 of the models to participant recall accuracy. This feature allows us to regress the generative accuracy of each of the three models
249 onto empirical recall accuracy per transfer trial and evaluate the R-squared of the regression as a goodness-of-fit measure.

250 The motif learning model ($R^2 = 0.34$) explains more variance of participants' transfer performance compared to the
251 chunking model ($R^2 = 0.04$), suggesting that projecting sequences in a projected motif space, an abstraction process, is critical
252 to capture human behavior in this task. The motif learning model also explains more variance than the he associative learning
253 model ($R^2 = 0.05$). Associative learning only is insufficient to capture participants' transfer behavior.

254 Experiment 2: Variable motifs

255 Experiment 2 tested the learning and transfer of variable motifs in the sequence recall paradigm. A training block of 40 trials
256 was followed by a transfer block of 24 trials. Participants were split into two groups: the variable group (motif) and the fixed
257 group (control). The variable group was instructed to remember sequences with variable motif B X D F, D X B F, F X D B (2).
258 X represents a variable and randomly assumes a letter amongst A, C, and E with equal probability with every occurrence. The
259 fixed group was instructed to remember unchanging sequences assuming the form: B A D F, D C B F, F E D B.

260 During the test block, both groups were instructed to remember a novel sequence with an embedded variable X: D X B F, F
261 X D B, B X F D. The location and entailment of X were the same as the training sequence with variables, but we changed the
262 fixed part of the sequence.

263 Model Prediction

264 We hypothesize that when participants are instructed to memorize sequences with a component that varies, identifying variable
265 entities and memorizing them in conjunction with the unvarying part of the sequence should facilitate transfer. That is, when
266 subjects encounter novel sequences sharing the same variable entity but different unvarying parts, they should memorize novel
267 sequences with overlapping variables better compared to the control group.

268 Training

269 **Behavioral Results** Figure 4 a shows the average sequence recall accuracy of the variable motif group and the fixed group. We
270 fitted participants' sequence recall accuracy with a linear mixed-effects regression model, assuming a by-participant random
271 intercept. The result showed a significant effect of group ($\chi^2(1) = 50.012, p \leq 0.001$). The fixed group recalled sequences
272 more accurately than the variable motif group ($\hat{\beta} = -0.22, se = 0.03, t(95) = -0.806, p \leq 0.001$). A changing part of the
273 instruction sequence hindered recall.

274 **Model Prediction** We trained the variable motif learning model on the same instruction sequences seen by participants.
275 For sequences with the variable motif, the model learned memory representation manifested in chunks and variables. To
276 do so, the model condensed observations of disparate instances of A, C, and E into one variable entity and concatenates the
277 variable entity with the already-acquired fixed sequence parts in its memory. In this way, the motif learning model learned to
278 represent instruction sequences with variable motifs as a chunk with embedded variable entities. Hence the memory contained
279 both concrete and abstract sequence parts as a low-complexity sequence representation. For control sequences, the model
280 constructed memory pieces by chunking. During recall, sampling entailment chunks of a variable entity introduces memory

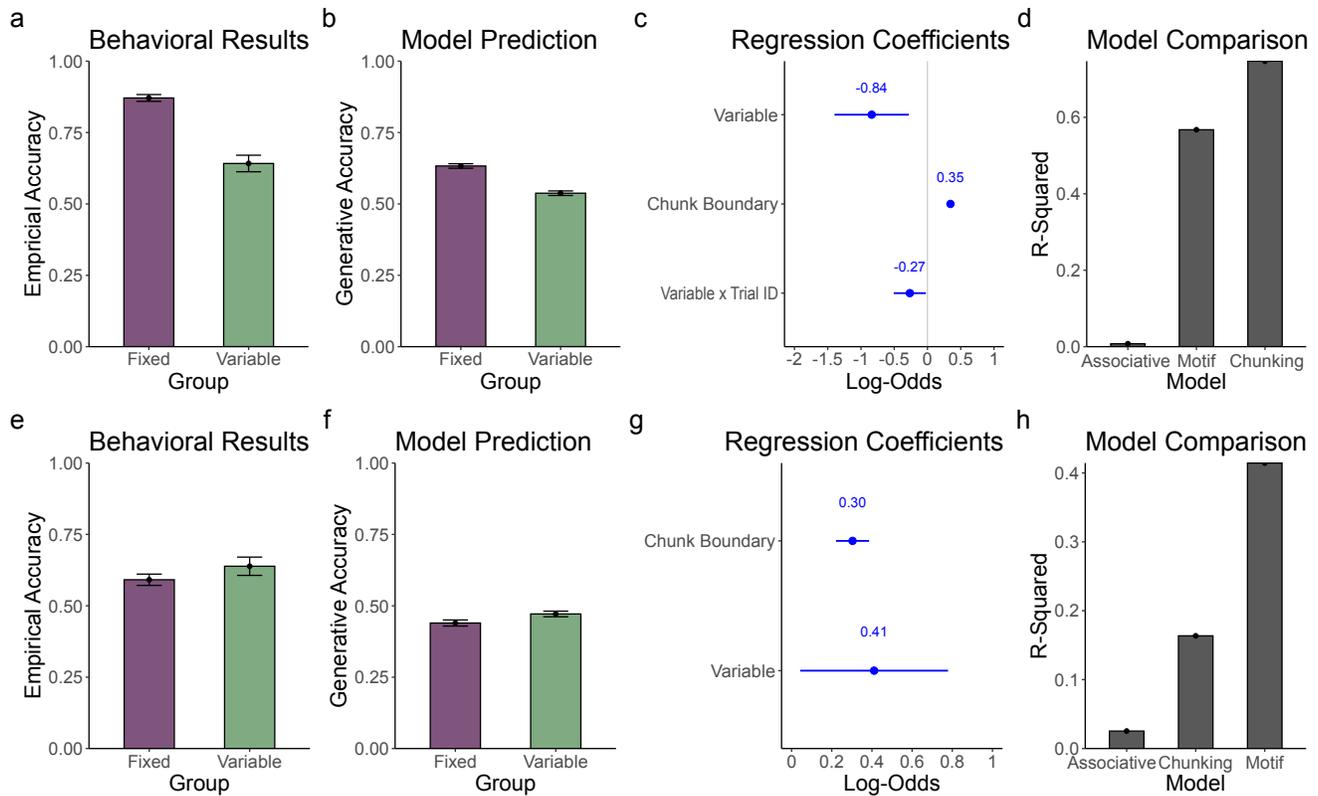


Figure 4. Model simulation and behavioral results for learning and transferring variable motifs a. Recall accuracy across groups during the training blocks. b. Simulated recall accuracy during the training blocks. c. Beta coefficient of a linear mixed effect logistic regression on recall key press correctness during the training blocks. d. Correlation between simulated generative accuracy and participants' recall accuracy. e. Recall accuracy across groups during the transfer blocks. f. Regression coefficients of logistic regression performed on recall keypress correctness during the transfer block. g. Correlation between simulated transfer generative accuracy and subjects' sequence recall accuracy. h. Correlation between training improvement (average recall accuracy difference between the last five training trials and the first five training trials) and the average recall accuracy during the initial 5 trials of the transfer block.

281 recall error ($\chi^2(1) = 54.37, p \leq 0.001$). The motif learning model recalled sequences with variable motifs less accurately than
282 fixed sequences ($\hat{\beta} = -0.09, se = 0.01, t(95) = -0.845, p \leq 0.001$).

283 **Regression Coefficient**

284 We then studied factors that influenced the keypress correctness via fitting a logistic mixed-effects regression, assuming
285 a per-participant random intercept and a random slope per serial position. Shown in Figure 4 c, the regression coefficient
286 suggested that the variable motif group was more prone to recall mistakes than the fixed group ($\beta = -0.84, se = 0.28,$
287 $z = -2.94, p \leq 0.003$). Apart from that, the variable group learned sequences slower than the fixed group ($beta = -0.26,$
288 $se = 0.12, z = -2.15, p = 0.03$). Training on sequences with variables decreased participants' probability of recalling the
289 correct key and slowed down learning. Overall, the regression analysis was consistent with our predictions.

290 **Model Comparison** We again compared the motif learning model with an associative learning model and a chunking
291 model by evaluating the R-squared value regressing simulation recall accuracy onto empirical recall accuracy. Figure 4 d shows
292 the goodness-of-fit model comparison on the training blocks.

293 The associative learning model ($R^2 = 0.008$) explained very little variance in subjects' recall accuracy progression during
294 learning, suggesting that just learning the first-order transition probability was insufficient to explain participants' learning
295 curve on memorizing sequences with variables. Having a chunking component that builds up recall memory pieces together
296 was essential to explain subjects' learning progression. Meanwhile, we observed that the chunking model ($R^2 = 0.74$) explained
297 more variance of recall accuracy progression than the variable learning model ($R^2 = 0.57$), possibly because the average
298 chunking process becomes more predictive of subjects' recall accuracy than the average variable learning process, as participants
299 may have learned variables in idiosyncratic ways that are not captured by the variable discovery process of the model but are
300 described better by a chunking model.

301 **Transfer**

302 **Behavioral Results** We hypothesized that participants transfer variable representations from the training to the test block.
303 Shown in Figure 3 e is the average recall accuracy of the two groups across all transfer trials. We used an independent-sample
304 t-test to assess the performance difference between the two groups, and a one-tailed t-test to assess the superiority of the variable
305 group compared to the fixed group in sequence recall. We observed a significant difference ($t(2119.6) = 4.14; p \leq 0.001;$
306 $95\%CI = [0.025, 0.071]$) in recall accuracy between the motif group ($M = 0.64$) and the control group ($M = 0.59$), supporting
307 our hypothesis that the variable group performs better at transfer than the fixed group.

308 **Model Prediction** As per model simulation shown in Figure 4 f, generative accuracy was higher for the model trained
309 on variable sequences than those trained on fixed sequences ($\hat{\beta} = 0.03, SE = 0.01, t(95) = 2.16, p \leq 0.03$) ($\chi^2(1) = 4.67,$
310 $p = 0.03$). This transfer advantage results from the variable learning model reusing the previously learned variables to parse
311 and chunk in conjunction with the novel sequence part. In other words, the model trained on sequences with variables learned
312 to ignore a certain part of the novel sequences to afford memorizing the unchanging sequence part.

313 **Regression Coefficients** We fitted a mixed-effect logistic regression on participants' recall key press correctness in the
314 transfer block, assuming a per-participant random intercept and a logit link function. Shown in Figure 4 g, we observed a
315 positive effect of train condition ($\beta = 0.41, se = 0.19, z = 2.19, p = 0.02$). Training on sequences with variable motifs helped
316 participants recall novel sequences sharing the same variable motif better than the control group trained on fixed sequences,
317 consistent with our model's prediction.

318 **Model Comparison** We compared the motif learning model with the chunking and associative learning model on the
319 transfer block. Shown in Figure 4 h, we observed that the motif learning model that reuses its previously learned variables to
320 memorize novel sequences explains the most human recall accuracy variance than the chunking and the associative learning
321 model. This aspect suggests that reusing previously learned variables to memorize novel sequences captures a part of the human
322 sequence memory variance when they transfer to novel sequences.

323 **Training Improvement Correlates with Transfer Performance** We also assessed the effect of training improvement on
324 transfer performance for both experimental groups. The improvement measure is evaluated on individual participants' sequence
325 average recall accuracy between the last five trials at the end of the training block, subtracted by the first five trials at the
326 beginning of the training block. This difference reflects the average improvement over the training period for every participant.
327 We observed a significant interaction between training improvement and group ($RSS = 2.44, F(1) = 10.42, p = 0.001$) affecting
328 transfer recall accuracy. Participants who improved more during training on variable motifs performed better during the initial
329 transfer blocks, compared to control ($\beta = 0.53, se = 0.17, t = 3.22, p = 0.002$). Training improvement on variable motifs
330 facilitated transfer to sequences sharing the same variables.

331 **Discussion**

332 We effortlessly perceive and extract motifs in music, acquire grammatical structure from languages, and use mathematical
333 variables to find out about the unknown. Already during early childhood, we can learn abstract concepts as soon as we learn
334 concrete concepts^{11,12}. Linguistics suggest that the conceptual metaphor —mapping similar structural concepts of a known

335 thing to construct an understanding of an unknown concept — plays a vital role in human understanding and reasoning^{13,14}.
336 Having seen a solution to a problem, people can solve problems in a similar conceptual relational space¹⁵. Abstraction as a
337 principle has demonstrated its usage in mathematics and machine learning. Mathematicians have used abstraction as a mapping
338 principle to transfer deductions from one formal system to a new formal system¹⁶. Abstraction has long been postulated as a
339 crucial requirement for intelligent agents to solve problems in diverse situations¹⁷. Reinforcement learning studies suggest
340 that state or action abstraction makes the representation more compact, easier to plan, and generalize flexibly to different
341 environments and across tasks^{18–22}. Yet, current artificial intelligence systems do not explicitly abstract in the way that humans
342 do²³. Hence, understanding how humans arrive at abstraction more generically has wide and profound implications in the study
343 of artificial and natural intelligence.

344 As the key to generalization, transfer, and planning, our ability to abstract from perceptual observations — which has
345 not received sufficient attention relative to its importance in intelligence — urges us to take a closer look at how abstraction
346 arises from sequential perceptual sequences. In the current work, we have proposed two specific sequence abstraction types:
347 projectional motifs — patterns derived from sequences through a projectional function, and variable motifs — patterns that
348 combine both concrete and variable elements. We studied the process of abstract motif learning in sequences, tested the learning
349 and transfer of both motifs in a sequence recall paradigm, and proposed a model that abstracts sequences to compress sequence
350 representations with projectional and variable motifs. We found that our model explained human behavior well.

351 Previously, associative learning models have been shown to explain the human grammaticity learning and judgements^{1,2,24,25}.
352 Our model comparison between associative learning and motif learning suggests that associative learning alone is insufficient
353 to explain human abstraction learning and transfer in sequence recall. As an alternative account of sequence learning, chunking
354 models including PARSER⁴, HCM⁸, CCN and TRACX^{6,7} acquire repeated patterns from sequences as chunks. Model
355 comparison between the chunking model and motif learning model suggests that the chunking model captures a part of variable
356 motif learning but not variable motif transfer, nor the learning and transfer of projectional motifs. Expanding the space of
357 chunking from concrete sequences to abstract spaces is vital to capture the motif learning and transfer effects observed in our
358 experiments.

359 Other works on sequence learning and mental compression include Planton et al.²⁶. In a binary auditory sequence violation
360 detection task, they showed that a language-of-thought model’s minimal description length of binary sequences relates to human
361 psychological complexity^{26,27}. Our work further relates mental compression with sequence motif learning. Rather than a static
362 account of sequence complexity, our model proposes a discovery process of actively learning sequence motifs during practice.

363 Limitations

364 Our work has limitations. In Experiment 1, learning one motif facilitated participants’ transfer to a different motif (3 e). The
365 same was not true for the model: learning one motif impaired its ability to transfer to the other different motif. The model’s
366 ability to recall a new motif is hindered when it has already learned one motif. This occurs because the recall process involves
367 sampling subsequences acquired since the start of training, and the previously learned chunks from the training motif may
368 still get sampled during the recall process which interferes with recall accuracy. This effect is consistent with the proactive
369 interference effect in the literature that memory for previously presented lists impairs memory for later presented lists^{28–31}. In
370 contrast, in our experiment, it seems as if humans are establishing a fresh context for structure discovery when encountering a
371 new motif^{32–34}. This phenomenon can be attributed to yet an additional layer of contextual abstraction that the model does not
372 capture. Namely, training on sequences with motifs guides people to look for motifs in subsequent sequences. Indeed, it has
373 also been observed in other tasks that structured training leads participants to look for structures in subsequent tasks³⁵. We
374 encourage future work to delineate the relation between memory interference effect and the facilitation of structural priors on
375 memory.

376 Additionally, most of our analysis compare model predictions with human behavior on an aggregated level. We encourage
377 future investigations to examine subjects’ idiosyncratic learning and transfer strategies. Apart from that, our work defines and
378 investigates two particular types of abstraction. We encourage future work to extend the investigation and look at more forms of
379 abstraction or automatic ways of discovering abstraction such as hierarchical clustering and chunking on recursive abstract
380 levels.

381 Conclusion

382 A vital role of abstraction is to facilitate sequence compression and generalization, and we proposed a motif learning model
383 based on this principle. Our model builds up a sequence memory via chunking motifs in an abstract space in search of a
384 low-complexity sequence representation, facilitating memorization and transfer. We developed a sequence recall task to
385 examine whether the two proposed motif types aid in learning and generalization. Our findings suggest that both motifs
386 facilitate sequence memorization and generalization to novel, unseen sequences. Humans showed similar behavior to the model
387 in learning and generalization of both abstraction types. This suggests that sequence compression via abstraction is a plausible

388 mechanism to explain human performance in sequence memory tasks. Our work paves the way for a better understanding of
389 how people construct abstract representations from observational sequences for efficient compression and transfer.

390 **Method**

391 **Ethics Statement**

392 Informed consent was obtained from all subjects before participation, and the experiments were performed following the
393 relevant guidelines and regulations approved by the ethics committee of the University of Tuebingen (Ethik-Kommission an
394 der Medizinischen Fakultät der Eberhard-Karls-Universität und am Universitätsklinikum Tübingen), under the study title:
395 Experimente zum Sequenz- und Belohnungslernen, with application number 701/2020BO.

396 Participants' data were analyzed anonymously. Upon agreement to participate in the study, they consented to a data
397 protection sheet approved by the data protection officer of the MPG (Datenschutzbeauftragte der MPG, Max-Planck-Gesellschaft
398 zur Förderung der Wissenschaften).

399 **Paradigm**

400 Specifically, six equally distanced squares are horizontally placed on the display. Each assumes a distinct color: blue, yellow,
401 magenta, red, green, and teal and corresponds to one legitimate key on the keyboard: S, D, F, J, K, and L. Participants were
402 instructed to place their fingers stationarily on these designated keys throughout the task (left index finger on D, left middle
403 finger on S, left ring finger on A, right index finger on J, right middle finger on K, and right ring finger on L). To control for
404 finger familiarity biases, a random mapping from keyboard position to color is generated for each participant.

405 Before the start of each trial, all colors were initially covered by dark shades. The sequence was then presented sequentially
406 by revealing each color for 800 ms followed by a brief re-covering of dark shades for another 200 ms before the next display
407 color. The colored sequence was presented in three groups of four, separated by pauses of 800 ms accompanied by the display
408 of a pair of paws, akin to the structure of a three-prose-poem with four words in each prose and pauses in between.

409 Following the sequence display, participants were prompted to recall the instructed sequence by pressing the corresponding
410 key. Upon the press of each key, the shade covering the corresponding color would disappear and the color would be revealed
411 for 200 ms. At the end of each group, a pair of paws would appear to signify the completion of one subsequence. At the end
412 of the third recall group, participants received immediate feedback on their recall accuracy and recall time which marks the
413 completion of one trial. Participants were instructed to prioritize both speed and accuracy and received a performance-based
414 bonus based on both factors. Before the official trials, participants completed a practice trial to familiarize themselves with the
415 task.

416 **Recruitment of Participants**

417 We recruited 135 participants for Experiment 1 from Prolific, an online crowd-sourcing experimental platform. Out of all
418 participants, thirty-seven were female. Participants' ages ranged from 18 to 67, with an average of 32 and a median of 28. The
419 experiment took an average of 45.06 minutes to complete. As compensation, participants received a base pay of £4 and another
420 performance-dependent bonus up to £4. The average hourly pay for the study was £11.60.

421 We recruited 120 participants for Experiment 2 from Prolific, out of which thirty-four were female. Participants' ages
422 ranged from 19 to 63, with an average of 31.2 and a median of 28. The experiment took an average of 47.55 minutes to
423 complete. As compensation, participants received a base pay of £4 and another performance-dependent bonus up to £4. The
424 average hourly pay for the study was £10.89.

425 **Payment**

426 For both experiments, participants receive feedback about their trial-wise bonus, which is dependent on a mixture of their
427 sequence recall accuracy and reaction time and is ceiled to the maximum bonus divided by the number of trials. The reaction
428 time bonus becomes the maximum when the recall reaction time is less than 2000 ms, and is set to 0 when the recall reaction
429 time exceeds 10000 ms. For reaction time in the middle, the bonusfast is calculated as $bonus_{fast} = bonus_{max} - (10000 -$
430 $trial_{rt}) / (10000 - 2000) \times max_{trial} bonus$. In this way, a reaction time between the two limits will yield a steady bonus increase.

431 The trial-wise bonus for accuracy is calculated as follows: when the recall accuracy is perfect, the bonusacc is set to
432 $max_{trial} bonus$. And when the recall accuracy is below 50%, which corresponds to more than 6 of the recalled sequences in a
433 false order or a false recalled item, then the bonusacc for this trial is set to 0. A recall accuracy in between will yield a bonusacc
434 calculated as $bonus_{acc} = bonus_{max} \times (trial_{acc} - 0.5) / (1 - 0.5)$.

435 Finally, the trial bonus is calculated as an average of the reaction time bonus and the recall accuracy bonus $trial_{bonus} =$
436 $0.5 \times bonus_{fast} + 0.5 \times bonus_{acc}$.

437 At the end of the experiment, trial-wise performance-dependent bonus was summed up to the total amount of bonus that
438 participants will receive.

439 Filtering

440 We applied the same filtering criteria on the training blocks for all groups as a basis to exclude participants: mean reaction time
441 $\leq 10,000$ ms (that is 10 seconds to press a sequence of 12 made of two distinct colors), mean recall accuracy $\geq 50\%$. On top of
442 that, we measured whether participants were learning by inspecting reaction time decrease, as a violation of a decrease in rt
443 would be an indication of distraction during the study. When applying a linear regression model regressing trial number on
444 reaction time on participant's data during the training blocks, the reaction time should on average, decrease, which translates to
445 having a significant ($p \leq 0.05$) of a negative beta coefficient. No filtering criteria were applied to the transfer blocks. After
446 filtering, 37 participants are left in group m1, 41 in m2, and 28 in group independent. The average accuracy was 0.80 ± 0.22 ,
447 and the average reaction time was $5446 \pm 3723(\text{std})$ ms.

448 For experiment 2, we excluded participants who took on average more than 20 seconds to recall a sequence during the
449 training block (since experiment 2 employs more colors than experiment 1, we also relaxed this exclusion criteria accordingly).
450 Since the motif condition is harder than the control condition, we applied different exclusion criteria for the two groups, and
451 excluded participants with an average sequence recall accuracy below 50% in the fixed group (as they have to recall the same
452 sequence repeatedly), and below 20% in the variable group. Additionally, we excluded people who do not have a significant
453 reaction time decrease ($p \leq 0.05$) during the training block — an indicator of not learning during the task. The exclusion criteria
454 apply only to the training blocks and no participants are excluded based on their transfer block performance. 23 participants
455 were excluded given that they have violated any of the above-mentioned criteria. After exclusion, 45 participants out of 120
456 remained in group m1, and 52 remained in group control. The average accuracy was 0.70 ± 0.28 , and the average reaction time
457 was 8094 ± 6209 ms.

458 **Sequence Recall** The model receives the same instruction sequences to participants as its training sequences, except that the
459 middle pauses were removed. To recall, the initial item of the sequence is used as a primer for the model to recall subsequent
460 sequential items. Based on the sequence segments stored in the model, it samples from the set of sequence segments that
461 are consistent with the sequence prime while giving priority to sampling larger segments. Once the first sequential segment
462 is sampled, the segment becomes the previous item to sample the next segment, which is based on the transition given the
463 occurrence of the previous segment. The recall complexity is evaluated by calculating the sampled probability of the recalled
464 sequence. $P(c_1, c_2, c_3) = P(c_1)P(c_2|c_1)(c_3|c_2)$, calculated from the marginal and conditional frequencies are both stored in the
465 model.

466 **Random Effect Structure of Regression Analysis** To obtain the maximal random effect structure justified by design
467 without inflating the Type I error rate³⁶, while balancing the loss of statistical power³⁷, we systematically select models across
468 multiple possible random effect structures and report the best model that is supported by data. Specifically, when fitting linear
469 mixed effect logistic regression on keypress correctness, we compared across random intercept per participant, random slope
470 per serial position, and trial ID, and always reported the best fitting model that includes any subset of the three random effects.

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474 Code and Data Availability Statement

475 The data collected and code used for analyzing this study can be found in this github repository: [https://github.com/
476 swu32/motif_learning](https://github.com/swu32/motif_learning)

477 Competing Interests

478 The authors have declared that there are no competing interests.

479 Author Contributions

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484 **Writing – original draft:** Shuchen Wu.

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References

1. Gomez, R. L. & Gerken, L. Artificial grammar learning by 1-year-olds leads to specific and abstract knowledge. *Cognition* **70**, 109–135, DOI: [10.1016/S0010-0277\(99\)00003-7](https://doi.org/10.1016/S0010-0277(99)00003-7) (1999).
2. Gómez, R. L. Variability and Detection of Invariant Structure. *Psychol. Sci.* **13**, 431–436, DOI: [10.1111/1467-9280.00476](https://doi.org/10.1111/1467-9280.00476) (2002). Publisher: SAGE Publications Inc.
3. Marcus, G. F., Vijayan, S., Bandi Rao, S. & Vishton, P. M. Rule learning by seven-month-old infants. *Sci. (New York, N.Y.)* **283**, 77–80, DOI: [10.1126/science.283.5398.77](https://doi.org/10.1126/science.283.5398.77) (1999).
4. Perruchet, P. & Vinter, A. Parser: A model for word segmentation. *J. Mem. Lang.* **39**, 246 – 263, DOI: <https://doi.org/10.1006/jmla.1998.2576> (1998).
5. Wu, S., Éltető, N., Dasgupta, I. & Schulz, E. Chunking as a rational solution to the speed–accuracy trade-off in a serial reaction time task. *Sci. Reports* **13**, 7680, DOI: [10.1038/s41598-023-31500-3](https://doi.org/10.1038/s41598-023-31500-3) (2023).
6. Servan-Schreiber, E. & Anderson, J. Learning artificial grammars with competitive chunking. *J. Exp. Psychol. Learn. Mem. Cogn.* **16**, 592–608, DOI: [10.1037/0278-7393.16.4.592](https://doi.org/10.1037/0278-7393.16.4.592) (1990).
7. French, R. M., Addyman, C. & Mareschal, D. TRACX: A recognition-based connectionist framework for sequence segmentation and chunk extraction. *Psychol. Rev.* **118**, 614–636, DOI: [10.1037/a0025255](https://doi.org/10.1037/a0025255) (2011).
8. Wu, S., Elteto, N., Dasgupta, I. & Schulz, E. Learning Structure from the Ground up—Hierarchical Representation Learning by Chunking. In Koyejo, S. *et al.* (eds.) *Advances in Neural Information Processing Systems*, vol. 35, 36706–36721 (Curran Associates, Inc., 2022).
9. Ebbinghaus, H. *Über das Gedächtnis: Untersuchungen zur experimentellen Psychologie* (Duncker Humblot Leipzig, 1885).
10. Lewandowsky, S. & Jr, B. Memory for serial order. *Psychol. Rev.* **96**, 25–57, DOI: [10.1037/0033-295X.96.1.25](https://doi.org/10.1037/0033-295X.96.1.25) (1989).
11. Ohlsson, S. & Lehtinen, E. Abstraction and the acquisition of complex ideas. *Int. J. Educ. Res.* **27**, 37–48, DOI: [10.1016/S0883-0355\(97\)88442-X](https://doi.org/10.1016/S0883-0355(97)88442-X) (1997).
12. Gentner, D. & Hoyos, C. Analogy and Abstraction. *Top. Cogn. Sci.* **9**, 672–693, DOI: [10.1111/tops.12278](https://doi.org/10.1111/tops.12278) (2017). *_eprint:* <https://onlinelibrary.wiley.com/doi/pdf/10.1111/tops.12278>.
13. Lawler, J. M. Metaphors we live by. *Language* **59**, 201–207 (1983).
14. Hofstadter, D. R. Analogy as the core of cognition. *The analogical mind: Perspectives from cognitive science* 499–538 (2001).
15. Duncker, K. On problem-solving. *Psychol. Monogr.* **58**, i–113, DOI: [10.1037/h0093599](https://doi.org/10.1037/h0093599) (1945).
16. Giunchiglia, F. & Walsh, T. A theory of abstraction. *Artif. Intell.* **57**, 323–389, DOI: [10.1016/0004-3702\(92\)90021-O](https://doi.org/10.1016/0004-3702(92)90021-O) (1992).
17. Konidakis, G. On the necessity of abstraction. *Curr. Opin. Behav. Sci.* **29**, 1–7, DOI: [10.1016/j.cobeha.2018.11.005](https://doi.org/10.1016/j.cobeha.2018.11.005) (2019).
18. Abel, D., Hershkovitz, D. E. & Littman, M. L. Near Optimal Behavior via Approximate State Abstraction. **9**.
19. Eckstein, M. K. & Collins, A. G. E. Computational evidence for hierarchically structured reinforcement learning in humans. *Proc. Natl. Acad. Sci.* **117**, 29381–29389, DOI: [10.1073/pnas.1912330117](https://doi.org/10.1073/pnas.1912330117) (2020).
20. Jetchev, N., Lang, T. & Toussaint, M. Learning grounded relational symbols from continuous data for abstract reasoning (2013).
21. Luciw, M. & Schmidhuber, J. Low complexity proto-value function learning from sensory observations with incremental slow feature analysis. In *Proceedings of the 22nd International Conference on Artificial Neural Networks and Machine Learning - Volume Part II, ICANN'12*, 279–287, DOI: [10.1007/978-3-642-33266-1_35](https://doi.org/10.1007/978-3-642-33266-1_35) (Springer-Verlag, Berlin, Heidelberg, 2012).
22. Silver, T. *et al.* Inventing relational state and action abstractions for effective and efficient bilevel planning. *ArXiv abs/2203.09634* (2022).
23. Chollet, F. On the measure of intelligence. *ArXiv abs/1911.01547* (2019).
24. Saffran, J. R., Newport, E. L. & Aslin, R. N. Word segmentation: The role of distributional cues. *J. Mem. Lang.* **35**, 606–621, DOI: <https://doi.org/10.1006/jmla.1996.0032> (1996).
25. Saffran, J. R., Johnson, E. K., Aslin, R. N. & Newport, E. L. Statistical learning of tone sequences by human infants and adults. *Cognition* **70**, 27–52, DOI: [https://doi.org/10.1016/S0010-0277\(98\)00075-4](https://doi.org/10.1016/S0010-0277(98)00075-4) (1999).

- 534 **26.** Planton, S. *et al.* A theory of memory for binary sequences: Evidence for a mental compression algorithm in humans.
 535 *PLOS Comput. Biol.* **17**, e1008598, DOI: [10.1371/journal.pcbi.1008598](https://doi.org/10.1371/journal.pcbi.1008598) (2021). Publisher: Public Library of Science.
- 536 **27.** Dehaene, S., Al Roumi, F., Lakretz, Y., Planton, S. & Sablé-Meyer, M. Symbols and mental programs: a hypothesis about
 537 human singularity. *Trends Cogn. Sci.* **26**, 751–766, DOI: [10.1016/j.tics.2022.06.010](https://doi.org/10.1016/j.tics.2022.06.010) (2022).
- 538 **28.** Wickens, D. D. Encoding categories of words: An empirical approach to meaning. *Psychol. Rev.* **77**, 1–15, DOI:
 539 [10.1037/h0028569](https://doi.org/10.1037/h0028569) (1970).
- 540 **29.** Wickens, D. D., Born, D. G. & Allen, C. K. Proactive inhibition and item similarity in short-term memory. *J. Verbal Learn.*
 541 *Verbal Behav.* **2**, 440–445, DOI: [10.1016/S0022-5371\(63\)80045-6](https://doi.org/10.1016/S0022-5371(63)80045-6) (1963).
- 542 **30.** Watkins, O. C. & Watkins, M. J. Buildup of Proactive Inhibition as a Cue-Overload Effect. .
- 543 **31.** Watkins, M. J. & Watkins, O. C. Cue-overload theory and the method of interpolated attributes. *Bull. Psychon. Soc.* **7**,
 544 289–291, DOI: [10.3758/BF03337192](https://doi.org/10.3758/BF03337192) (1976).
- 545 **32.** Dennis, S. & Humphreys, M. A context noise model of episodic word recognition. *Psychol. review* **108**, 452–78, DOI:
 546 [10.1037/0033-295X.108.2.452](https://doi.org/10.1037/0033-295X.108.2.452) (2001).
- 547 **33.** Farrell, S. Temporal clustering and sequencing in short-term memory and episodic memory. *Psychol. Rev.* **119**, 223–271,
 548 DOI: [10.1037/a0027371](https://doi.org/10.1037/a0027371) (2012).
- 549 **34.** Brown, G., Neath, I. & Chater, N. A temporal ratio model of memory. *Psychol. review* **114**, 539–76, DOI: [10.1037/](https://doi.org/10.1037/0033-295X.114.3.539)
 550 [0033-295X.114.3.539](https://doi.org/10.1037/0033-295X.114.3.539) (2007).
- 551 **35.** Schulz, E., Franklin, N. T. & Gershman, S. J. Finding structure in multi-armed bandits. *Cogn. Psychol.* **119**, 101261, DOI:
 552 <https://doi.org/10.1016/j.cogpsych.2019.101261> (2020).
- 553 **36.** Barr, D. J., Levy, R., Scheepers, C. & Tily, H. J. Random effects structure for confirmatory hypothesis testing: Keep it
 554 maximal. *J. Mem. Lang.* **68**, 255–278, DOI: <https://doi.org/10.1016/j.jml.2012.11.001> (2013).
- 555 **37.** Matuschek, H., Kliegl, R., Vasishth, S., Baayen, H. & Bates, D. Balancing type i error and power in linear mixed models.
 556 *J. Mem. Lang.* **94**, 305–315, DOI: <https://doi.org/10.1016/j.jml.2017.01.001> (2017).
- 557 **38.** Oberauer, K. Understanding serial position curves in short-term recognition and recall. *J. Mem. Lang.* **49**, 469–483, DOI:
 558 [https://doi.org/10.1016/S0749-596X\(03\)00080-9](https://doi.org/10.1016/S0749-596X(03)00080-9) (2003).
- 559 **39.** Cowan, N., Saults, J., Elliott, E. M. & Moreno, M. V. Deconfounding Serial Recall. *J. Mem. Lang.* **46**, 153–177, DOI:
 560 [10.1006/jmla.2001.2805](https://doi.org/10.1006/jmla.2001.2805) (2002).

561 **Supplementary Information**

562 **0.1 Experiment 1**

563 **0.1.1 Training**

564 **Regression Coefficient** Other regressors that showed significant effects are serial position, trial ID, chunk boundary, and the
 565 number of repetitions. Serial position is the n-th item recalled in a trial, significantly affecting recall correctness ($\chi^2(1) = 697.92$,
 566 $p \leq 0.001$). The further the position of a sequence recall, the more likely that participants will be making a mistake ($\beta = -0.31$,
 567 $se = 0.03$, $z = -10.82$, $p \leq 0.001$). This result is consistent with the primacy effect widely observed in the serial recall
 568 literature³⁸ as mistake probability increases with the serial position. Apart from that, trial ID, i.e., the number of practice trials
 569 ($\chi^2(1) = 810.02$, $p \leq 0.001$), also increases the log-odds of recalling correctly ($\beta = 0.09$, $se = 0.02$, $z = 4.45$, $p \leq 0.001$),
 570 confirming a practice effect over training blocks.

571 We also observed that the sub-sequence boundary (at the first, fourth, fifth, eighth, ninth, and twelfth item of the sequence)
 572 affects recall correctness ($\chi^2(1) = 34.767$, $p \leq 0.001$). Items located at the beginning and the end of the displayed sub-
 573 sequence are more likely to be recalled correctly compared to the items within each sub-sequence ($\beta = 0.15$, $se = 0.02$,
 574 $z = 6.01$, $p \leq 0.001$). This observation resonates with the literature suggesting subjects have more accurate memory and
 575 recall performance at the boundaries of serially ordered sub-sequences than between^{10,33,39}. Additionally, the number of exact
 576 repetitions ($\chi^2(1) = 158.19$, $p \leq 0.01$) increases the log odds of correct recall press ($\beta = 0.07$, $se = 0.01$, $z = 5.10$, $p \leq 0.001$).

577 **0.1.2 Transfer**

578 **Regression Coefficient** Apart from transfer types, the recall keypress correctness decreases with the recall sequence position
 579 ($\chi^2(1) = 322.3$, $p \leq 0.001$). The further subjects are into recall, the more likely they will make mistakes ($\beta = -0.08$, $\sigma = 0.008$,
 580 $z = -10.78$, $p \leq 0.001$). The decrease in recall accuracy is consistent with the recency effect in memory literature: items that
 581 occur early in a sequence tend to be remembered and recalled more accurately³⁸. We also observed a practice effect: trial ID
 582 affects the log odd ratio of pressing the right key ($\chi^2(1) = 86.07$, $p \leq 0.001$) ($\beta = 0.09$, $se = 0.01$, $z = 5.94$, $p \leq 0.001$). Apart

583 from that, chunk boundary effect was also observed: the subchunk boundaries generally exhibit a higher recall accuracy than
584 the interchunk items ($\chi^2(1) = 25.17, p \leq 0.001$) ($\beta = 0.17, \sigma = 0.03, z = 5.17, p \leq 0.001$), resonating with existing findings
585 that chunk boundaries are remembered more accurately than within-chunk items³³.

586 **0.2 Experiment 2**

587 **0.2.1 Training**

588 **Regression Coefficient** Other regressors that showed significant effects are serial position ($\beta = -0.67, se = 0.04, z = -15.25,$
589 $p \leq 0.001$), confirming the recency effect; Trial ID ($\beta = 0.53, se = 0.14, z = -15.24, p \leq 0.001$), confirming the practice effect;
590 the number of repetitions ($\beta = 0.05, se = 0.01, z = 4.71, p < 0.001$); and chunk boundary ($\beta = 0.34, se = 0.02, z = 13.29,$
591 $p \leq 0.001$).

592 **0.2.2 Transfer**

593 **Regression Coefficient** Similar to the training block, we observed a recency effect ($\beta = -0.64, se = 0.02, z = -29.1,$
594 $p < 2e - 16$), practice effect ($\beta = 0.32, se = 0.02, z = 12.78, p \leq 0.001$), repetition effect. ($\beta = 0.08, se = 0.03, z = 2.51,$
595 $p \leq 0.001$), and chunk boundary effect ($\beta = 0.30, se = 0.04, z = 7.23, p \leq 0.001$), confirming a viable expectation over
596 experimental manipulation.