Describing and Controlling Multivariate Nonlinear Dynamics:

A Boolean Network Approach

Xiao Yang¹, Nilam Ram¹, Peter C. M. Molenaar², Pamela M. Cole²

¹Stanford University

²The Pennsylvania State University

Author Note

Xiao Yang, Department of Psychology, Stanford University; Nilam Ram, Departments of Psychology and Communication, Stanford University; Peter C. M. Molenaar, Department of Human Development and Family Studies, The Pennsylvania State University; Pamela M. Cole, Department of Psychology, The Pennsylvania State University

Address correspondence regarding this manuscript to: Xiao Yang, Department of Psychology, Stanford University, 450 Jane Stanford Way, Building 420, Stanford University, Stanford, CA 94305 , USA or xfy5031@stanford.edu.

Acknowledgements

Thanks very much to the study participants and families for providing a detailed glimpse of their emotional dynamics, and to the many research assistants who manually code the data. This work was supported by the National Institute on Health (R01 HD076994, T32 AG049676) and the Penn State Social Science Research Institute.

Abstract

We introduce a discrete-time dynamical system method, the Boolean network method, that may be useful for modeling, studying, and controlling nonlinear dynamics in multivariate systems, particularly when binary time-series are available. We introduce the method in three steps: inference of the temporal relations as Boolean functions, extraction of attractors and assignment of desirability based on domain knowledge, and design of network control to direct a psychological system toward a desired attractor. To demonstrate how the Boolean network can describe and prescribe control for emotion regulation dynamics, we applied this method to data from a study of how children use bidding to an adult and/or distraction to regulate their anger during a frustrating task ($N = 120$, $T = 480$ seconds). Network control strategies were designed to move the child into attractors where anger is OFF. The sample shows heterogeneous emotion regulation dynamics across children in 22 distinct Boolean networks, and heterogeneous control strategies regarding which behavior to perturb and how to perturb it. The Boolean network method provides a novel method to describe nonlinear dynamics in multivariate psychological systems and is a method with potential to eventually inform the design of interventions that can guide those systems toward desired goals.

Keywords: Boolean network, network control, psychological systems, nonlinear dynamics

(Word Count: 10,762)

Describing and Controlling Multivariate Nonlinear Dynamics: A Boolean Network Approach

This paper introduces and forwards a Boolean network approach that can be used to model and describe nonlinear psychological dynamics from multivariate binary time-series, and as a foundation for designing and enacting network control measures that are likely to keep the dynamical system in desired states. In brief, and as will be explained in greater depth later, a Boolean network $G(\mathbf{X}, \mathbf{B})$ consists of a set of nodes that are binary variables $\mathbf{X} = \{x_1, \dots, x_n\}$ and edges that are Boolean functions $\mathbf{B} = \{f_1, \dots, f_n\}$. The Boolean functions connect nodes together using Boolean operators “AND”, “OR”, and “NOT”, the specifics of which are inferred through algorithmic search so that they provide the best prediction for each outcome node/variable. Once the network is identified, typically occurring desirable and undesirable attractor states (as determined by content experts), can then be extracted through permutation of the inferred Boolean functions across all possible initial conditions. Together, the knowledge of the network and the attractors allows researchers to design control strategies that might be used to direct the system out of undesired attractors and toward desirable attractors. After outlining the motivation for and details of the Boolean network approach, we illustrate the utility of the Boolean network method for study of within-person dynamics using multivariate binary time-series data ($T=480$ seconds) obtained in a study of children’s emotion regulation ($N = 120$ 36-month-olds). Our example illustrates how the Boolean network method can be used to identify behavior that (dys)regulates each child’s anger, and provide a template for how personalized advice might be generated for improving a child’s emotion regulation. Our presentation provides a novel method for studying psychological dynamics, analyzing the plethora of binary time-series data collected in psychological science, and designing control for multivariate psychological systems.

Motivating Problem: Development of Emotion Regulation

Emotion regulation is conceptualized as a *dynamic, interactive process* between prepotent responses and executive processes (Cole, Ram, & English, 2018). Prepotent responses (PR) are the highly probable actions that take priority over other response options (Arnold, 1960), for example, frustration about a blocked goal and intensified effort to overcome the obstacle (e.g., pounding on a vending machine for a jammed snack). Executive processes (EP) are cognitive processes – attention, memory, language, and reasoning – that are invoked when engaging strategies meant to change whether and how a prepotent response is enacted (Cole et al., 2018), for example, shifting attention away from a desired object (e.g., focusing on what a person is saying during a meeting, rather than the chocolate donut sitting on the table). Emotion regulation, then, can be defined as the *influence* of strategies that recruit the executing capacities of human cognition to alter the intrinsic dynamics of prepotent response (Cole et al., 2018; see also Cole, Martin, & Dennis, 2004; Kopp, 1989). In other words, emotion regulation is located in the temporal dynamics between PR and EP.

From a developmental perspective, emotion regulation dynamics emerge and become refined with age (Cole et al., 2018; Morales et al., 2018). By the time they enter school, children are expected to have sufficient regulatory skills to delay their own goals while focusing on teacher-defined activities (Cole, Tan, Hall, Zhang, Crnic, Blair, & Li, 2011). For example, children must learn to regulate emotions like frustration and disappointment when they must wait for a desired object or fun activity. Ideally, the temporal dynamics of anger (PR) and attention focus (EP) during such a task might be described using simple statements such as: when attention is shifted *away* from a desired object (e.g., a toy that they are told to wait to open), then anger is reduced; when attention is *focused* on the desired object, then anger is perpetuated

(Gilliom, Shaw, Beck, Schonberg, & Lukon, 2002; Benson, Ram, & Stifter, 2018; Ramsook, Benson, Ram, & Cole, 2020). Evidence of successful emotion regulation entails a dynamic process wherein negative emotions are altered by the use of strategies that have potential to modify prepotent responses when they are needed. As they develop, children are expected to *learn to use* behaviors that shift attention *away* from the desired object (e.g., distraction) in order to reduce anger (desirable attractor) and *learn to avoid* behaviors that shift attention *toward* the desired object (e.g., bidding about the object) and perpetuate anger (undesirable attractor). The *developmental or long-term learning goal* is not that children erase their negative emotions, but that they can, when needed, *effectively* engage executive control and enact *behaviors* that reduce negative emotions.

Binary Time-Series and Methods to Describe and Control Multivariate Dynamical System

Studies of children and adults often make use of laboratory-based experimental/observational research paradigms (e.g., delay of gratification; Mischel, Shoda, & Rodriguez, 1989). In these studies, behavioral data are very often produced through video coding procedures wherein moment-to-moment presence or absence of specific behaviors (e.g., emotions and strategies; Cole et al., 2011) are coded as ON or OFF. These studies have produced an abundance of binary time-series data. We (and others) want to use these data to describe and understand a wide variety of phenomena, including children's emotion regulation.

To *describe* emotion regulation dynamics using the readily available data, we need methods that allow us to model the temporal dynamics in binary time-series. A few of the available methods have been applied in psychology, including the Ising model (Ising, 1925) and a Markov chain method (Meyn & Tweedie, 2005). The Ising model is a network-based method focused on describing the dynamics of a system of binary nodes using node-specific activation thresholds and node-pairwise interactions. Psychologists have used the Ising model to study psychopathology and describe the (de)activation of and reinforcing relations among multiple symptoms (van Borkulo et al., 2014; Cramer et al., 2016; Epskamp, Maris, Wardorp, & Borsboom, 2018). The Markov chain method is a time-series-based method focused on describing the probability that individuals will, from one time point to the next, transition from one multivariate state to another. Psychologists have used more advanced variations of the Markov chain method such as reciprocal Markov modeling to study progression of individuals' mood states and food consumption (Lu, Pan, Zhang, Dube, & Ip, 2015), and mixed Markov modeling to study the relation between trait-level neuroticism and day-level negative affect states (de Haan-Rietdijk, Kuppens, Bergeman, Sheeber, Allen, & Hamaker, 2017). In this paper, we forward the Boolean network approach as another alternative.

Besides providing an additional way to model temporal dynamics in binary time-series data, the Boolean network approach directly supports the design of control strategies that might be used to direct the system out of undesired attractors and toward desirable attractors. For example, caregivers may want to help children *achieve* their developmental goals (e.g., emotion regulation skills) by providing them with some personalized guidance on which strategies (EP) can be used when to effectively modify their undesirable emotions (PR). The Boolean network approach provides detailed information about what specific guidance might be most helpful.

Control theory (or control system design), a subfield of mathematics and engineering, focuses on moving dynamic systems toward desired goals. A variety of mathematical tools and algorithms have been developed to determine the specific actions (control input) that will guide the behavior of a dynamical system so that it follows a desired trajectory or settles into a desired state (Lewis, Vrabie, & Syrmos, 2012; Liu & Barabasi, 2016). Using these tools and algorithms a

variety of control systems have been developed to, for example, adjust engine thrust and steering in real-time so that planes land successfully in a changing environment (e.g., windy conditions).

Psychology researchers have, for decades, proposed to use control theory in psychological systems (Carver & Scheier, 1998; Molenaar & Ram, 2010; Molenaar & Nesselrode, 2015), and there have been a few advanced applications of linear control theory to psychological systems. Molenaar (1987) proposed a feedback control method for optimal control of psychotherapeutic processes, where a patient's time-varying psychological dynamics (e.g., emotional tone, self-esteem, social activities, physical fitness) are estimated session by session, and the optimal control input performed by the therapist (e.g., counseling and modeling, positive/negative reinforcement, interactive style) is computed subsequently. Even though the data were fitted retrospectively, the proposed optimal control method can be applied in therapeutic settings to guide therapists' on-going practice. More recently, Sinclair and Molenaar (2008) introduced an advanced optimal control technique, receding horizon feedback-feedforward control, to identify and prescribe the optimal amount of intervention (e.g., insulin input) that would keep patients functioning within desirable bounds (e.g., normal range blood glucose levels). This receding horizon feedback-feedforward control method, with time-varying parameters estimated in a state-space modeling framework using Extended Kalman filtering with Iteration and Smoothing (EKFIS) to and fed into a recursive optimal controller that minimizes fluctuations, has since been applied with success to estimate and control glucose levels in Type I diabetes patients (Wang et al., 2014). Other recent empirical work creatively uses a network controllability metric to inform therapists selection of symptom(s) as intervention targets, and a linear quadratic regulator (LQR) to evaluate the efficacy of specific interventions (Henry, Robinaugh, & Fried, 2020). Despite the interest and advancement of control theory methods, the empirical applications of control in psychology remains scarce and almost absent for guidance of *nonlinear* psychological systems. In this paper, we suggest that the Boolean network method can be used to describe and control nonlinear dynamic systems in psychological science and practice.

Boolean Network

The Boolean network (BN) method, originally introduced by Kauffman (1969, 1993) to study gene regulatory networks, has been used extensively in systems biology to model and prescribe control for nonlinear dynamical systems (Cheng & Qi 2009; Zañudo & Albert, 2015). A Boolean network $G(X(t), B)$ is defined by a set of nodes $X(t) = \{x_1(t), x_2(t), \dots, x_n(t)\}$, where x_i is the i^{th} node and $x_i(t)$ is the observed time-series of the i^{th} node, and a set of Boolean functions $B = \{f_1, f_2, \dots, f_n\}$, where each Boolean function $f_i(x_{i1}, x_{i2}, \dots, x_{ik})$ with k specific input nodes for node x_i is used to determine the state of x_i at time $t+1$. When $x_1(t) = 1$, the node is in an activated state at time t (i.e., ON, behavior occurring), and when $x_1(t) = 0$, the node is dormant at time t (i.e., OFF, behavior not occurring). The vector formed by the current state of all $i = 1$ to n nodes, $X(t)$, describes the current state of the system, and the set of Boolean functions, B , describe the dynamics of the system, how the states change from $X(t)$ to $X(t + 1)$.

The Boolean functions represent the temporal dynamics (i.e., how the nodes are influenced by other nodes) using three Boolean operators, AND $\wedge$, OR $\vee$, and NOT $\bar{x}$ that when combined together can express complex relations. The AND ($\wedge$) operator is defined as *all* input variables must be ON to turn the outcome ON; the OR ($\vee$) operator is defined as *any* input variables being ON will turn the outcome ON; the NOT ($\bar{x}$) operator simply defines the opposite state of the input variable. Table 1 shows how these rules produce different outcomes based on the input of two variables.

The nonlinear dynamics in a Boolean network are described using the Boolean operators. When a Boolean function contains two variables connected with an AND operator, it represents a multiplicative relation between the two input variables. The function $z = x \text{ AND } y$ is similar to $z = x \times y$ because only when $x = y = 1$ will $z = 1$. This kind of multiplicative relation is a *nonlinear* relation between the input variables and the outcome variable. Given that the AND operator and OR operator can be transformed to each other, the OR operator can also capture nonlinear dynamics. And, when the AND operator is combined with the NOT operator, all propositional first-order logic of nonlinear dynamics can be described (Whitesitt, 2010).

To give an intuitive illustration of how a Boolean network works, we use a two-node network consisting of x_1 and x_2 . The observed time-series of x_1 and x_2 are shown in Figure 1a. From this observed binary time-series, we can infer the Boolean functions

$$\begin{aligned}x_1(t+1) &= x_1(t) \wedge \overline{x_2(t)} \\x_2(t+1) &= x_2(t)\end{aligned}$$

(the details of how rules are inferred will be outlined later in the Boolean Network Implementation section). The first function, $x_1(t+1) = x_1(t) \wedge \overline{x_2(t)}$, indicates that x_1 at $t+1$ depends on both x_1 and x_2 at t . The AND operator coupled with the NOT operator imply that if x_1 is ON and x_2 is also ON at t , then x_1 will turn OFF at $t+1$. The second function, $x_2(t+1) = x_2(t)$, indicates that the state of x_2 at $t+1$ only depends on the state of x_2 at t . If x_2 is ON at t , then x_2 will continue to be ON at $t+1$.

The network graph of these Boolean functions is shown in Figure 1b. The first function, $x_1(t+1) = x_1(t) \wedge \overline{x_2(t)}$, is expressed using an expanded Boolean network diagram (Wang & Albert, 2011) where the AND operator is expressed using an additional composite node structure. For our example, the first function is made explicit by the inclusion of the composite node along with a green edge from x_1 to the composite node and a red edge from x_2 to the composite node that indicates the positive and negative (i.e., NOT) relations within the AND operator. In turn, the green edge pointing from the composite node to x_1 indicates the equal relation in the original function. The second function, $x_2(t+1) = x_2(t)$, is indicated by a green edge from x_2 to itself. Similar to other modeling frameworks (e.g., SEM), the diagram maps directly to the Boolean algebra.

From an applied perspective, we can interpret this simple example Boolean network as a description of a child's emotion regulation dynamics. Suppose x_1 and x_2 represent a child's expression of anger and use of distraction during each second of a laboratory wait task. From the first Boolean function, we know that anger (x_1) is regulated by distraction because whenever distraction (x_2) is ON at t , anger (x_1) will be turned OFF at $t+1$. From the second Boolean function, we know that distraction (x_2) is not activated by anger (x_1) because anger is not anywhere in the function. The Boolean network thus provides a dynamical system model that can be used to *describe* the dynamics embedded in the binary time-series data collected in many laboratory-based studies of emotion regulation.

Attractors and Their Desirability

Once the temporal dynamics are described by Boolean functions, attractors can be extracted based on the dynamics. The purpose of extracting attractors is to articulate the persistent patterns individuals might get stuck in. Researchers can then use domain knowledge and practical concerns to evaluate the (un)desirability of each attractor and determine whether to intervene. For example, in our empirical example on children's emotion regulation, persistent anger qualifies as an undesirable attractor. If a child gets stuck in this kind of attractor, the child might need some guidance on how to get out of their persistent anger state.

Attractors of a dynamical system indicate the long-term behavior of the system, specifically, an area of the phase space – the set of possible states of the system – that the system occupies or approaches more frequently than other areas (Carver & Scheier, 2002). If a dynamical system is thought of as a physical object whose movement is governed by the dynamical rules, then an attractor is where the dynamical system prefers to stay in this physical space (Thelen & Smith, 2005). There are many types of attractors, including fixed-points, limit cycles, saddles, cusps, and so on. A fixed-point attractor means the system tends toward a single fixed value, e.g., a pendulum *with* friction eventually returns to a stable point. In contrast, a limit cycle attractor means the system tends to cycle through a set of states with different values, e.g., a pendulum *without* friction oscillates. As well, and especially important in the modeling of psychological systems, nonlinear dynamical systems can have multiple attractors and be *multi-stable*, in that they spend long periods in multiple areas of the phase space (Boker & Graham, 1998). Following other work on the derivation of attractors in Boolean networks (Kauffman, 1993; Aracena, Gomez, & Salinas, 2013), we will, in our empirical illustration, identify a collection of fixed-point, limit cycle, and multiple attractor cases.

The desirability of attractors may be derived from domain-specific knowledge. For example, cancerous states (Campbell & Albert, 2019), smoking behavior (Boker & Graham, 1998), and activated depression states (Cramer et al., 2016) have been deemed as undesirable attractors. Sometimes the desirability of attractors changes according to the developmental context. For example, crawling is an attractor of locomotion that may be desirable for younger children that have not yet developed the ability to walk but be undesirable when children are older and expected to walk (Thelen & Smith, 2005).

Continuing with the two-node Boolean network in Figure 1 as an illustrative example, we demonstrate that the Boolean network method can extract the multi-stability of a dynamical system using a state transition graph (the details of how the derivation works will be introduced in the Data Analysis section). To illustrate, we represent all the possible states of x_1 and x_2 as tuples (x_1, x_2) . For example, the tuple (0,0) indicates the state where both x_1 and x_2 are OFF. The state-to-state transitions implied by the Boolean functions can then be depicted as a state transition graph, as shown in Figure 1c for the Boolean network shown in Figure 1b. Following the Boolean functions, we can see that the system will transition from (1,1) to (0,1), and then from (0,1) to (0,1), and so on. The (0,1) state is deemed a fixed-point attractor because the system will not transition to any other states once it is in (0,1). We can also see in the state transition graph that there are two other fixed-point attractors (0,0) and (1,0).

Once extracted, the attractors can be interpreted with respect to domain knowledge. For example, consider the case where x_1 and x_2 represent a child's anger and distraction, respectively, during a frustrating wait. Following the definitions surrounding emotion regulation that were given above, the three fixed-point attractor states can be classified as desirable or undesirable. The hope for most parents and teachers is that the child will not express persistent anger. Thus, (0,0) and (0,1) are considered desirable attractors because anger is OFF, and (1,0) is considered an undesirable attractor because anger is ON. The next goal, of course, is to direct the child towards those desirable non-anger attractor states.

Boolean Network Control and Emotion Regulation

The Boolean network method utilizes the differential desirability of attractors *within the same dynamical system* to design network control strategies that might facilitate movement out of undesirable attractors and toward desirable attractors. In brief, specific control strategies are derived by determining the shortest distance between the undesirable attractor and another state

that will lead to or is a desirable attractor and suggesting a new Boolean function that will systematically “push” the system along the shortest distance (the details of how derivation works will be introduced in the Data Analysis section). Again using the two-node Boolean network as an illustrative example, the researcher’s objective is to design a network control strategy that eliminates undesirable attractors. As shown in Figure 1d, one network control strategy is to perturb x_2 when x_1 is ON. The system will thus transition from the undesirable attractor state (1,0) to (1,1), and then from (1,1) to the desirable attractor (0,1).

Interpretation with respect to control of a child’s emotion regulation dynamics is relatively straightforward. A new strategy is introduced to the child: *If anger (x_1) is ON, then turn distraction (x_2) ON*. When the child follows this new rule, anger will eventually be turned OFF. In real-world applications, if parents and teachers guide children to develop this control strategy, the child may eventually be able to, on their own, effectively use behaviors as regulatory strategies to reduce negative emotions when they occur.

Boolean Network Implementation

In this section, we review the essential concepts and methodological details of the Boolean network method. We synthesize methodological presentations from the systems biology literature into a psychology-oriented implementation with three major steps: (1) inference of the temporal relations among multiple binary variables as Boolean functions, and construction of a Boolean network in which the binary variables are nodes and the Boolean functions are edges, (2) extraction of attractors based on the inferred dynamics and assignment of desirability for each attractor based on domain knowledge, and (3) design of network control to direct the system toward a desired attractor by identifying how the Boolean network needs to be updated.

Inference of Boolean Functions and Construction of Boolean Networks

The Boolean functions can be inferred from the observed time-series of all the variables using a variety of algorithms (Lähdesmäki, Shmulevich, & Yli-Harja, 2003). Simulation studies provide good evidence that these algorithms can accurately recover the key features of continuous-scale dynamics (e.g., oscillation) in a variety of settings (Berestovsky & Nakhleh, 2013). In practice, the Boolean functions can be inferred from data and the Boolean network representation of the dynamic system constructed in R using the BoolNet package (Müssel, Hopfensitz, & Kestler, 2010). An implementation tutorial is included in the Supplementary Materials that accompany this paper and at <https://quantdev.stanford.edu>.

To illustrate how the inference proceeds, we describe a brute force Boolean computation algorithm. Boolean functions can vary in their complexity, from simple functions where the value of the outcome $x_i(t + 1)$ is given by a constant to complex functions where the value of $x_i(t + 1)$ is informed by many input variables. In any given model, the size of the network is denoted as N and the number of input variables allowed into the inferred Boolean functions is denoted as K . When K is low, all the functions are simple. As K increases the functions may get more and more complicated and may thus better capture the intricate dependencies embedded in a complex system. Given that N is often fixed a-priori by the research question or data availability, our goal in the model construction phase is to identify the best fitting and most parsimonious network. Like the estimating procedure of ordinary least squares, the identification of Boolean functions is done by comparing the fitted value and the observed value and finding the Boolean function that produces the minimal error. The Boolean functions are inferred for a network where $K = 1$, where $K = 2$, where $K = 3$, and so on, and model fits are compared to select the model with optimal fit versus parsimony trade-off (more on this below).

Inference when $K = 1$. When $K = 1$, the optimal set of Boolean functions is inferred by comparing the observed time-series of each outcome variable $x_i(t + 1)$ with time-series constructed using Boolean functions where the outcome is determined by either a constant or a function of *one* input variable, $x_{i1}(t)$, which can be any variable in the system, including the variable itself at the previous timepoint, namely $x_i(t)$. Specifically, we cycle through all possible constant and unary functions

$$\begin{aligned} \text{CONSTANT: } x_i(t + 1) &= 0 \\ x_i(t + 1) &= 1 \\ \text{UNARY: } x_i(t + 1) &= x_{i1}(t) \\ x_i(t + 1) &= \overline{x_{i1}(t)} \end{aligned}$$

and select the function that has minimal error, where error is calculated for each variable as the sum of all discrepancies at every timestep across the entire time-series (i.e., the sum of all false positives *and* false negatives; false positives are where $x_i(t + 1)$ is predicted to = 1 but was actually = 0, and false negatives are where $x_i(t + 1)$ is predicted to = 0 but was actually = 1, Lähdesmäki et al., 2003). The search for the optimal set of $K = 1$ functions is done straightforwardly and quickly using brute force.

Inference when $K = 2$. When $K = 2$, it gets a little more complicated. The Boolean functions are inferred by comparing the time-series of the outcome variable $x_i(t + 1)$ with time-series constructed using Boolean functions where the outcome is determined by *two* input variables, $x_{i1}(t)$ and $x_{i2}(t)$ (meaning any pair of variables in the system that are used to estimate $x_i(t + 1)$, that are combined using any combination of the Boolean operators (AND, OR, NOT). Search for the best fitting Boolean functions is conducted in the following order:

$$\begin{aligned} \text{AND: } x_{i1} \wedge x_{i2}, x_{i1} \wedge \overline{x_{i2}}, \overline{x_{i1}} \wedge x_{i2}, \overline{x_{i1}} \wedge \overline{x_{i2}} \\ \text{OR: } x_{i1} \vee x_{i2}, x_{i1} \vee \overline{x_{i2}}, \overline{x_{i1}} \vee x_{i2}, \overline{x_{i1}} \vee \overline{x_{i2}} \\ \text{XOR: } x_{i1} \oplus x_{i2}, x_{i1} \oplus \overline{x_{i2}}, \overline{x_{i1}} \oplus x_{i2}, \overline{x_{i1}} \oplus \overline{x_{i2}} \end{aligned}$$

where XOR is a succinct way to describe the rule $x_{i1} \oplus x_{i2} = \overline{x_{i1}} \wedge x_{i2} \vee (x_{i1} \wedge \overline{x_{i2}})$, and means that the outcome = 1 only when the two input variables have opposite values.

Starting with the inference of AND rules, the essence of the algorithm is to compute the time series that are obtained through the AND operation on all possible pairs of two variables in the system at time t and the outcome variable at time $t+1$, and then *select* the pair whose product of the AND operator matches the outcome variable time-series most closely. To illustrate the inference process of $x_i(t + 1) = x_{i1}(t) \wedge x_{i2}(t)$, we infer the Boolean function for $x_1(t + 1)$

$$\text{from an observed multivariate binary times-series } (x_1, x_2, x_3) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 0 \end{pmatrix} \text{ where } t = 1 \text{ to } 5.$$

The time series of the outcome $x_1(t + 1)$ is $\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ where $t = 1$ to 4, thus $t + 1 = 2$ to 5. Each

possible pairs of two variables in the system at time t ($t = 1$ to 4), connected with AND rule, can

$$\text{then be computed as a new time-series, e.g., } x_1(t) \wedge x_2(t) = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \wedge 1 \\ 1 \wedge 0 \\ 1 \wedge 1 \\ 0 \wedge 1 \end{pmatrix} =$$

$$\begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \end{pmatrix}, x_1(t) \wedge x_3(t) = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 0 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \wedge 0 \\ 1 \wedge 1 \\ 1 \wedge 0 \\ 0 \wedge 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}, x_2(t) \wedge x_3(t) = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix} \wedge \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \wedge 0 \\ 0 \wedge 1 \\ 1 \wedge 0 \\ 1 \wedge 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \text{ We can compare the time-series of the outcome variable and the two-}$$

variable-pairs connected by AND rule, and compute the error, e.g., $x_1(t+1)$ compared with $x_1(t) \wedge x_2(t)$ has error = 3, because the values at $t = 1$ and 2 are false negatives and $t = 3$ is a false positive, $x_1(t+1)$ compared with $x_1(t) \wedge x_3(t)$ has error = 1, because the values at $t = 1$ is a false negative, $x_1(t+1)$ compared with $x_2(t) \wedge x_3(t)$ has error = 3, because the values at $t = 1$ and 2 are false negatives and $t = 4$ is a false positive. Hence, $x_1 \wedge x_3$ is selected as the best fit. The other three possible Boolean functions with AND rules, namely $x_{i1}(t) \wedge \overline{x_{i2}(t)}$, $x_{i1}(t) \wedge \overline{x_{i2}(t)}$, $\overline{x_{i1}(t)} \wedge x_{i2}(t)$, can be searched by the same procedure after applying the NOT operator to one or both of the input variables.

The inference of OR rules proceeds in a similar manner as the AND rules, by comparing the time series that are obtained through the OR operation on all possible pairs of two variables in the system at time t and the outcome variable at time $t+1$, and then *select* the pair whose product matches the outcome variable time-series most closely.

Inference of the XOR rules is similar. First note that $x_i(t+1) = x_{i1}(t) \oplus x_{i2}(t)$ if $x_{i1}(t+1) \oplus x_i(t) = x_{i2}(t)$. So to find the $x_{i2}(t)$ that satisfies $x_i(t+1) = x_{i1}(t) \oplus x_{i2}(t)$, we simply compare the time-series of $x_{i1}(t+1) \oplus x_i(t)$ with the time series of every $x_{i2}(t)$, and the best matching time-series is the $x_{i2}(t)$ that satisfies $x_i(t+1) = x_{i1}(t) \oplus x_{i2}(t)$. The inference of the other three possible Boolean functions with XOR rules, namely $x_{i1}(t) \oplus \overline{x_{i2}(t)}$, $\overline{x_{i1}(t)} \oplus x_{i2}(t)$, $\overline{x_{i1}(t)} \oplus \overline{x_{i2}(t)}$, will be searched by the same procedure after applying the NOT operator to one or both of the input variables. Since $x_{i1}(t) \oplus \overline{x_{i2}(t)} = \overline{x_{i1}(t)} \oplus x_{i2}(t)$ and $\overline{x_{i1}(t)} \oplus x_{i2}(t) = \overline{x_{i1}(t)} \oplus \overline{x_{i2}(t)}$, we only need to compare the time-series of $x_{i1}(t) \oplus \overline{x_{i2}(t)}$ with $x_i(t+1)$ for the rest three Boolean functions with XOR rule.

By comparing the time-series of the outcome variable $x_i(t+1)$ with time-series constructed using all possible combinations of *two* input variables, $x_{i1}(t)$ and $x_{i2}(t)$ and the above Boolean functions using Boolean operators (AND, OR, XOR), each variable pair combined by each form of Boolean function will have an error, which is the sum of false positives and false negatives over the entire time-series. We then can select the input variables and the Boolean function form with the minimal error as the inferred Boolean function of $x_i(t+1)$.

Inference when $K > 2$. When $K > 2$, the inference can be made recursively with the input of two variables as the foundation. In the case of $K \geq 2$, a recursive algorithm is used to build the inference upon the case of $K = 2$. Let us consider a case of three input variables. We can infer the Boolean functions $f(x_{i1}, x_{i2}, x_{i3})$ in the following way. First, infer the rules of two input variables, x_{i1}, x_{i2} , denoted as $f_1(x_{i1}, x_{i2})$, that matches the value of the outcome $x_i(t+1)$ when $x_{i3}(t) = 1$, and then infer the rules of two input variables, denoted as $f_2(x_{i1}, x_{i2})$, that matches the value of the outcome $x_i(t+1)$ when $x_{i3}(t) = 0$.

The inferred Boolean function is as follows:

$$f(x_{i1}, x_{i2}, x_{i3}) = (x_{i3} \wedge f_1(x_{i1}, x_{i2})) \vee (\overline{x_{i3}} \wedge f_2(x_{i1}, x_{i2})) \quad (1)$$

Here is the proof. When $x_{i3}(t) = 1$, Equation 1 can be transformed in the following equation:

$$\begin{aligned}
f(x_{i1}, x_{i2}, x_{i3}) &= (1 \wedge f_1(x_{i1}, x_{i2})) \vee (0 \wedge f_2(x_{i1}, x_{i2})) \\
&= (1 \wedge f_1(x_{i1}, x_{i2})) = f_1(x_{i1}, x_{i2})
\end{aligned} \tag{2}$$

Since we inferred $f_1(x_{i1}, x_{i2})$ by matching to the outcome $x_i(t+1)$, when $x_{i3}(t) = 1$, thus $f(x_{i1}, x_{i2}, x_{i3}) = x_i(t+1)$. Similarly, we consider when $x_{i3}(t) = 0$, Equation 1 can be transformed as the following equation:

$$\begin{aligned}
f(x_{i1}, x_{i2}, x_{i3}) &= (0 \wedge f_1(x_{i1}, x_{i2})) \vee (1 \wedge f_2(x_{i1}, x_{i2})) \\
&= 1 \wedge f_2(x_{i1}, x_{i2}) = f_2(x_{i1}, x_{i2})
\end{aligned} \tag{3}$$

given that $x_i(t+1) = f_2(x_i, x_j)$ when $x_{i3}(t) = 0$, $f(x_{i1}, x_{i2}, x_{i3}) = x_i(t+1)$. Note here f_1 and f_2 have to have the same input variables x_{i1}, x_{i2} , so the number of input variables K is kept at 3 after adding x_{i3} .

Any higher number of input variables could be inferred by a similar principle recursively. For example, when the number of the input variable is 4, we can infer the Boolean functions $f(x_{i1}, x_{i2}, x_{i3}, x_{i4})$ for a fixed x_{i4} in the following way. First, infer the rules of three input variables, denoted as $f_1(x_{i1}, x_{i2}, x_{i3})$, that matches the value of the outcome $x_i(t+1)$ when $x_{i4}(t) = 1$, and then infer the rules of three input variables, denoted as $f_2(x_{i1}, x_{i2}, x_{i3})$, that matches the value of the outcome $x_i(t+1)$ when $x_{i4}(t) = 0$. The new rules are the different combinations of f_1 and f_2 in the form of $(x_{i4} \wedge f_1(x_{i1}, x_{i2}, x_{i3})) \vee (\overline{x_{i4}} \wedge f_2(x_{i1}, x_{i2}, x_{i3}))$.

For each $K = 1, 2, 3, \dots$ a complete Boolean network is obtained by putting all the inferred Boolean functions together. The above is a brute force Boolean function computation method, a more time-efficient algorithm using the fingerprint function and fast matrix multiplication is introduced by Akutsu and colleagues (Akutsu, Miyano, & Kuhara, 2000).

Identification of K . As can be seen from the exposition on how the Boolean functions are inferred, increases in K introduce lots of additional complexity and possibility. Indeed, already when $K > 2$, the Boolean functions can describe systems with aperiodic attractors that is sensitive to initial conditions, which means a small difference in the initial condition will turn into divergent trajectories over time (e.g., like the Lorenz system). Those complexities will make it very difficult for researchers to determine the (un)desirability of attractors and/or use the Boolean networks to design viable control strategies. Thus, it is often useful to put an upper bound on K (denoted as $maxK$) before searching for Boolean functions in order to avoid situations where the search algorithm becomes intractable (Akutsu, et al. 2000). In the absence of strong theory about network complexities, we suggest doing analysis with a $maxK = 2$. Although there is some tradeoff in overall fit, this will at least keep the attractors and subsequent network control interpretable. As more theory and knowledge about the nonlinearities in the system emerge, other choices may be preferred. In our forthcoming example, we identify the final optimal model by comparing fits and parsimony of networks obtained where $K = 1$, and $K = 2$ (or on up to $maxK$) and select the model with the lowest error.

Extraction of Attractors

After the Boolean functions are inferred, a state transition graph can be constructed by exhaustive search of all the state transition sequences that proceed from every permutation of initial conditions. The attractors are identified by constructing the state transition graph and finding the absorbing states (states that transition back to themselves).

We demonstrate this procedure using the two-node network introduced above in Figure 1,

$$\begin{aligned}
x_1(t+1) &= x_1(t) \wedge \overline{x_2(t)} \\
x_2(t+1) &= x_2(t)
\end{aligned} \tag{4}$$

There are $2^N = 2^2 = 4$ possible initial conditions (x_1, x_2) for the 2-node network, (0,0), (0,1), (1,0), (1,1). From each initial condition, we use the set of inferred Boolean functions $\mathbf{B}$ to compute the state of the system at all subsequent time steps. For example, when the system starts with the initial condition $(x_1=0, x_2=0)$ at $t=1$, then $x_1(t=2)=x_1(t=1) \wedge \overline{x_2(t=1)}=0 \wedge \overline{0}=0 \wedge 1=0$, and $x_2(t=2)=x_2(t=1)=0$. We then know (0,0) will transition to (0,0). Similarly, we can compute the subsequent state of the system for all the other initial conditions: (0,1), (1,0), and (1,1). The collection of state transitions is shown in Table 2.

The state transitions described by the Boolean functions can be drawn as a state transition graph, as shown in Figure 1c. All the possible states of the system, (x_1, x_2) , are represented as circles, and the arrows indicate what state will follow at time $t+1$ when the system is in a particular state at time t . The arrows that point back to the same state indicate the absorbing states where once the system is in that state, it will remain in that state. Here, (0,0), (0,1), and (1,0) are all transitioning back to themselves, and thus are the attractors of this dynamical system. Sometimes, the system will cycle through a limited set of states in a limit cycle or complex attractor.

Design of Network Control

The purpose of network control is to direct the system from undesirable attractors to desirable attractors (Shmulevich & Dougherty, 2010; Campbell & Albert, 2019). In our substantive example, we use network control to suggest what strategies can modify each individual child's undesirable behavior with implications for future ways to help children develop regulatory skills and not get stuck in negative emotions.

Network control in this paper focuses on modifying the state space transition graph in a Boolean network. The output of control system design should include three pieces of information: (1) which node(s) or Boolean function(s) needs to be changed, (2) the destination node(s) – the specific strategy of network control, and (3) what is the condition to change – the context of deploying network control. These three pieces of information will be useful to drive the system into more desirable attractors.

The general procedure for identifying a control strategy is as follows:

1. *Formulate the goal of network control in terms of the desirable attractor(s) and undesirable attractor(s).* Then, the goal of network control is to move the system out of the undesirable attractor and into the desirable attractor. The assignment of desirability to each attractor could be done based on domain knowledge and practical concerns.
2. *Compute the Hamming distance between the undesirable attractor and the states in a desirable attractor basin.* An *attractor basin* is defined as the set of states that will eventually go to the same attractor. We consider moving the system from an undesirable attractor to the basin of a desirable attractor because the system will eventually go to the desirable attractor. The distance from the undesirable attractor to each state in the desirable basin can then inform which nodes *need to be manipulated*. The distance here is computed using Hamming distance (Hamming, 1950), where we compare two states digit by digit, and then sum the number of digits that differ from each other as the distance. For example, the state (0,1) has a distance of 2 from the state (1,0) because the first and second digits differ.
3. *Identify the control strategy.* Suppose the undesirable attractor has a Hamming distance of 1 to a state in the desirable attractor basin, and the 1 difference is caused by node x_i . Suppose x_i is OFF in the undesirable attractor. Then “turning node x_i ON when the system is in the undesirable attractor” is our *control strategy*. There could be multiple

states that have a distance of 1 to the undesirable attractor, indicating multiple control strategies. It is also possible there is no state that has a distance of 1 to the undesirable attractor, indicating more nodes need to be manipulated to control the system.

4. *Formulate the new Boolean function for node x_i .* Once the specific node, say x_i , that needs to be perturbed is identified, the new form of this node's function should obey the following equations

$$f_{x_i} = \begin{cases} f_{x_i}, & \text{if } (x_{i1}, x_{i2}) \neq \text{undesirable attractor state} \\ \overline{f_{x_i}}, & \text{if } (x_{i1}, x_{i2}) = \text{undesirable attractor state} \end{cases} \quad (5)$$

So far, we have introduced how to identify which node to perturb, and under what circumstances we should perturb it. We now use an example to demonstrate the network control with Boolean functions and state transition graph before and after network control, shown in Figure 2. The original Boolean functions are in Equation 4. We know the attractors based on the state transition graph are (0,0), (1,0), (0,1) from Figure 2b. When $x_1 = 1$ is defined as undesirable, then (1,0) is an undesirable attractor and (0,0) and (0,1) are desirable attractors. Therefore, the control strategy is to perturb x_2 when the system is in the state (1,0), because the distance from (1,0) to (1,1) is 1, and (1,1) belongs to the desirable attractor basin. The new function of f_{x_2} is as follows:

$$f_{x_2} = \begin{cases} x_2, & \text{if } (x_1, x_2) \neq (1,0) \\ \overline{x_2}, & \text{if } (x_1, x_2) = (1,0) \end{cases} \quad (6)$$

We can also reduce the right-hand side of Equation 6 to one line. The function under specific condition can be expressed as a product of the function and the condition with an AND operator, e.g., $x_2, \text{ if } (x_1, x_2) \neq (1,0)$ can be expressed as $x_2 \wedge (x_1 \wedge \overline{x_2})$. The two conditions can be combined by an OR operator, e.g., $f_{x_2} = (x_2 \wedge (x_1 \wedge \overline{x_2})) \vee (\overline{x_2} \wedge (x_1 \wedge \overline{x_2})) = x_2(t) \vee (x_1(t) \wedge \overline{x_2(t)})$. Comparing this new Boolean function with the original inferred Boolean function of x_2 , we see that $x_1 \wedge \overline{x_2}$ is added to the original function with an OR operator, and this function means to turn x_2 ON when x_1 is ON and when x_2 is OFF. To confirm the new Boolean function can direct the system from an undesirable attractor to a desirable attractor basin, we plot the state transition graph after perturbation in Figure 3d.

In sum, we have introduced the Boolean network method and synthesized the implementation of the method into three steps, (1) inference of the Boolean functions and construction of the Boolean network; (2) extraction of attractors; and (3) design of network control that can direct the system into the researcher-defined desirable attractors. Following these three steps, we can describe the dynamics underlying multivariate (binary) time-series data as a Boolean network and identify network control strategies that can move the system into desirable states. In principle, the network control strategy is the basis for designing behavior modification and other intervention methods that can help individuals achieve desirable developmental goals.

Empirical Example

We illustrate how the Boolean network method can be used to examine emotion regulation dynamics and support the development of effective emotion regulation. This is accomplished by using data from a study of how children regulate their emotions during a frustration-inducing wait task at age 36 months. Comprehensive information about the

Development of Toddlers Study can be found in Cole et al. (2011). Details relevant to the present study are given here.

Participants

The analysis sample consisted of 117 children (64 boys) described by their mother as White (92%) or biracial (8%) (see also Cole et al., 2011 for complete demographic information). All procedures were approved by The Pennsylvania State University's Institutional Review Board (protocol #18993 and #45013).

Procedure

We examined the dynamics of children's emotion regulation in the context of a frustration-inducing wait task (Vaughn, Kopp, & Krakow, 1984). During the task, the child and mother were seated in an observation room. A research assistant provided the child with one boring toy (e.g., broken car) and a gift-wrapped in shiny paper, and the mother with questionnaires to complete. The children were instructed by their mother to wait until she finishes her work to open the gift. Mothers were instructed in advance that the experiment was to see how children tolerate waiting and to do whatever they ordinarily do when their child must wait for something the child wants. After 8 minutes, the research assistant returned, and the mother let the child open the gift.

Measures

The procedure was video recorded, and the videos were later coded for children's emotion expressions and behavioral strategic attempts. Here, we examine the dynamics of a specific negative emotion, *anger*, and two strategies, *bidding to mother about the demands of waiting* and *focused distraction*.

Anger expressions. Children's anger expressions were coded second by second using a system based on facial expressions and vocal quality (Cole, Zahn-Waxler, & Smith, 1994). Anger intensity was coded for each second on a 0 to 4 scale (0 = *not present*, 1 = *low intensity*, 2 = *moderate intensity*, 3 = *high intensity*). Interrater reliability was good (Cohen's $\kappa = .86$). For the purpose of demonstrating the Boolean network method, we simplified the codes as binary data, 0 = 0, and 1 = all ratings > 0.

Strategy use. Two strategies that children commonly use in these tasks were of interest. *Bids to mother* were defined by children's seeking the mother's attention about the demands of the wait (e.g., asking how much longer the wait is). *Focused distraction* was defined by children shifting attention away from the gift and becoming absorbed in an alternate, appropriate activity, such as playing. Interrater reliability for strategies was good (Cohen's $\kappa = .84$). Like anger, bids and focused distraction were coded as binary variables that indicated whether the behavior had not (=0) or had (=1) occurred in each second.

Results

We will introduce the results in two parts. The first part uses analysis of one child's data to illustrate how the three implementation steps are applied and interpreted. The second part summarizes the analysis of all 117 individuals in the sample.

Person-Specific Analysis: Identifying a Behavior Modification Strategy

For each of the 117 children, we followed the procedures outlined above: (1) inferred the Boolean functions and constructed the Boolean network, (2) extracted the attractors, and (3) designed network control strategy.

Inference of Boolean functions and construction of Boolean networks. Using the BoolNet package in R (Müssel et al., 2010), we constructed 3-node person-specific Boolean networks from each child's trivariate binary time-series (anger, bidding, and distraction). As

described earlier, the best fitting Boolean functions are inferred using recursive algorithms that test all possible combinations of Boolean operators to identify the set of functions that best fit the data (Akutsu et al., 2000). These functions, together with the nodes, form the Boolean network of the given child.

As an example, one child's binary time-series data and the inferred Boolean functions are shown in Figure 3a and 3b. The inferred Boolean functions are as follows:

$$anger(t + 1) = anger(t) \wedge \overline{distraction(t)} \quad (7)$$

$$bidding(t + 1) = bidding(t) \quad (8)$$

$$distraction(t + 1) = distraction(t) \quad (9)$$

Interpretation follows the earlier example. From Equation 7 we see that for this child, distraction regulates anger; if $distraction(t)$ is ON, meaning $\overline{distraction(t)}$ is OFF, then $anger(t + 1)$ will be OFF. From Equations 8 and 9 we see that, for this child, bidding is independent of anger and distraction, and that distraction is independent of anger and bidding. That means that this child's use of bidding or distraction is not activated by their anger state.

Extraction of attractors. For each child, we then used the person-specific Boolean networks to make a state transition graph and identify attractors – the states that transition back to themselves. By using domain knowledge, specifically that the general developmental goal is for children to be able to wait without expressing persistent anger, we formally defined attractor states where anger = ON as undesirable attractors and attractor states where anger = OFF as desirable attractors.

The state transition graph for our example child is shown in Figure 3c, where each dot represents specific states of the system as 3-tuples of the form (anger, bidding, distraction). Six attractors are identified, and each attractor basin is highlighted by a different color. The desirable attractors where anger is OFF are (0,0,0), (0,1,0), (0,0,1), (0,1,1), and the undesirable attractors where anger is ON are (1,0,0) and (1,1,0).

Design of network control. For network control design, we search for control strategies using the algorithm introduced earlier in the Boolean Network Implementation section. We also constrained the control strategy so that only the state of the behavior variable can be changed, because changing anger ON/OFF is considered only achievable via strategy use (the behavior variable). For our example child, one behavior modification strategy is found that will move the system from an undesirable attractor (1,0,0) to a desirable attractor (0,0,1) and from an undesirable attractor (1,1,0) to a desirable attractor (0,1,1). In more complicated cases, each undesirable attractor may have a different control strategy, and multiple control strategies may be found. In terms of updating the Boolean functions, a new Boolean function of distraction, produced by the design of network control, is shown in Figure 3d. This new Boolean function of distraction means, in addition to the original Boolean function $distraction(t)$, distraction needs to be modified in such a way that it takes the opposite state of distraction at t when anger is ON, which is expressed as $anger(t) \wedge \overline{distraction(t)}$. Putting this behavior modification with the original Boolean function together, it forms the updated Boolean function of distraction: $distraction(t) \vee anger(t) \wedge \overline{distraction(t)}$. Figure 3e shows the updated state transition graph. With the updated Boolean functions, we can see in Figure 3e that the system moves from the state (1,0,0) to a state (1,0,1), and then to a desirable attractor (0,0,1) (marked in green). The transition from the state (1,0,0) to a state (1,0,1) is enabled by taking the opposite state of distraction when anger is ON. With the new Boolean function introduced, all the attractors, (0,0,0), (0,0,1), (0,1,0) and (0,1,1) are desirable with the system always moving towards an anger = OFF state.

Between-Person Differences: Different Network, Attractors, and Control Strategies

Following the person-specific approach, after conducting analysis for each of the 117 children separately, we summarize across the sample and examine between-person differences. The full pattern of results is shown in Table 3 and Figure 4. Of note, although there are not yet any clear guidelines for a cutoff of acceptable error from the literature on inference of Boolean functions (Akutsu et al. 2000), the empirical distribution of error in our sample suggests that the obtained Boolean networks provide a good representation of the data. The error for Boolean function inference in the observations (480 seconds) was relatively low: the median of error of all three variables was 14 ($min = 0$, $max = 100$); for anger, the median of error was 9 ($min = 0$, $max = 84$); for bidding, the median of error was 23 ($min = 2$, $max = 100$); and for distraction, the median of error was 11 ($min = 0$, $max = 39$). Higher error implies there could be other sources of variation influencing the outcome variable (e.g., behaviors other than bidding or distraction).

Across the 117 children, we found 22 unique Boolean networks. These are shown in Table 3, organized with respect to their unique Boolean function expressions, attractor states, and optimal control strategies. The corresponding state transition graphs are shown in Figure 4. Networks with similar properties and control strategies are grouped together. In rows 1 to 4 of Table 3, there are 40 children whose anger is predicted to be 0/OFF with different functions for bidding and/or distraction, and all the attractors have anger = 0/OFF (in Figure 4a-d); hence no control strategy is needed. In rows 5 to 9, there are eight children whose *bidding is regulating anger* with different functions for bidding and/or distraction. The regulating effect of bidding is indicated by the Boolean function: $anger(t+1) = anger(t) \text{ AND NOT } bid(t)$. These networks have both desirable and undesirable attractors except one (in Figure 4e-i except 4g), and the corresponding control strategy is to *turn bidding ON when anger is ON*. There is one network (row 7 in Table 3, Figure 4g) that does not have a control strategy available. This is because bidding is 0/OFF in the Boolean function inference; therefore, there is no bidding = ON in the state transition graph. The search algorithm for the control strategy cannot find a state transition to turn bidding ON. In rows 10 to 12, there are seven children whose *distraction is regulating anger* with different functions for bidding and distraction. The regulating effect of distraction is indicated by the Boolean function: $anger(t+1) = anger(t) \text{ AND NOT } distraction(t)$. These networks have both desirable and undesirable attractors (in Figure 4j-l), and the corresponding control strategy is to *turn distraction ON when anger is ON*. In row 13, there are nine children whose *bidding is dysregulating anger*. The dysregulating effect of bidding is indicated by the Boolean function: $anger(t+1) = anger(t) \text{ AND } bidding(t)$. These networks have both desirable and undesirable attractors (in Figure 4m), and the corresponding control strategy is to *turn bidding OFF when anger is ON*. In rows 14 to 15, there are two children whose *distraction is dysregulating anger* with different functions for bidding and distraction. The dysregulating effect of distraction is indicated by the Boolean function: $anger(t+1) = anger(t) \text{ AND } distraction(t)$. One network has both desirable and undesirable attractors (in Figure 4n), and the corresponding control strategy is to *turn distraction OFF when anger is ON*. The other network has only a desirable attractor (in Figure 4o), so no control strategy is needed. Finally, in rows 16 to 22, there are 53 children whose *anger is not affected by bidding or distraction* with different functions for bidding and distraction. The no-effect on anger is indicated by the Boolean function: $anger(t+1) = anger(t)$. These networks have both desirable and undesirable attractors (in Figure 4p-v), but there is no control strategy available because anger is not affected by either of the behavior, so modifying either behavior will not turn anger OFF.

Substantive Interpretations. We highlight four important aspects of the results. First, there is striking *heterogeneity* within the sample in terms of the Boolean networks and their associated attractors and control design. One network can differ from another network in at least one Boolean function for one node (e.g., row 1 versus row 2), or they can differ by all three Boolean functions for all three nodes (e.g., row 4 versus row 5). Correspondingly, the state transition graph shows heterogeneous trajectories of each child's emotion system in the binary state space. Depending on the initial state of a system, the system can end up in one attractor (Figure 4d), two attractors (Figure 4c), three attractors (Figure 4a, f), four attractors (Figure 4b, g, h, o, r, t), five attractors (Figure 4k, l, n), six attractors (Figure 4e, i, j, m, p, s, u, v), and eight attractors (Figure 4q). Individual differences in children's emotion regulation (Diaz & Eisenberg, 2015) are most often based on either parent reports of children's traits or on observations using aggregated variables (e.g., sums). The Boolean approach has the potential to yield a nuanced analysis of within-person dynamics that might then improve our ability to conceptualize between-person differences.

Second, the control strategy depends on the regulatory dynamics between behavior (bidding or distraction) and anger. Two main configurations emerge. When a behavior, whether distraction or bidding, is regulating anger, the control strategy is to turn that behavior ON when anger is ON. In contrast, when a behavior, whether distraction or bidding, is dysregulating anger, the control strategy is to turn that behavior OFF when anger is ON. The implication is that there is not a one-size-fits-all control strategy. The specifics need to be matched to the actual dynamics of that child's system.

Third, not every network is controllable. The lack of controllability of the Boolean networks in this empirical example occurs when a child's anger is not being affected by any behavior. The independence of anger means that any control strategy based on changes in strategy use (the only place we can intervene) will be ineffective. As previously mentioned, we also constrained the control strategy so that only the state of the behavior variable can be changed, because changing anger ON/OFF is considered only achievable via strategy use (the behavior variable).

Fourth, from a substantive perspective, we can classify the networks with respect to the overall pattern of implied regulation ability in a way that propels future testing of a developmental hypothesis. Specifically, we can consider children as exhibiting one of three patterns of behavior: *well-developed*, *developing*, and *underdeveloped*. The *well-developed* pattern is referring to the networks that do not need control strategy (row 1 to row 4 and row 15 in Table 3, totally 40 children). The *developing* pattern is referring to the networks with available control strategy (rows 5 to 6 and 8 to 14 in Table 3, totally 24 children). The *underdeveloped* pattern is referring to the networks with no available control strategy (row 7 and rows 16 to 22 in Table 3, totally 53 children).

Discussion

This paper introduces the Boolean network method as a method to describe and prescribe control strategy for nonlinear, multivariate psychological dynamics. This method provides a novel method that can assess person-specific developmental goals and inform designs of prevention or intervention programs to improve development.

A Novel Method to Describe Nonlinear Dynamics

In this paper, we introduced the Boolean network method as a method for describing nonlinear dynamics. Specifically, system dynamics embedded in binary multivariate time series data are described using a set of Boolean functions that make use of AND, OR, and NOT

operators. This approach is complementary to other methods commonly used to model dynamics with binary time-series data – the Ising model and the Markov Chain method.

The first and foremost difference between the Boolean network model and the Ising model is that the mathematical expression of dynamics is different. A Boolean function that contains two input variables connected with an AND operator represents a multiplicative relation between the two input variables (formally, the Boolean algebra function $z = x \text{ AND } y$ is similar to the algebraic function $z = x \times y$). In both cases, these functions capture a *nonlinear* relation between the input variables and the outcome variable. The Ising model uses an exponential function, similar to logistic regression, to link the input variables/nodes and the outcome/node x_j (van Borkulo et al., 2014)

$$P(x_{j,t+1}|x_{\setminus j,t}) = \frac{\exp [\tau_j x_j + x_j \sum_{k \in V \setminus j} \beta_{jk} x_k]}{1 + \exp [\tau_j x_j + x_j \sum_{k \in V \setminus j} \beta_{jk} x_k]} \quad (10)$$

where x_j represent the j^{th} node in a network, τ_j is the threshold of activation for node x_j , and β_{jk} is the *pairwise interaction* (also called weight) between x_j and another node x_k (Note: $\setminus j$ indicates all indices but j). In the Ising model, the nonlinear dynamics are expressed in the pairwise interactions.

The Boolean network method also has a strong similarity with the Markov chain method, and the unique contribution of the Boolean network method is it provides explicit information about the dynamics between variables. The Boolean functions identify the information of regulatory dynamics. As a contrast, the Markov Chain method only focuses on estimating the state transition probability from t to $t+1$ (Meyn & Tweedie, 2005). The state transition in the Markov chain is similar to the state transition graph in the Boolean network method. Even though the dynamics between variables are embedded in the state transition probability in the Markov chain method, but the information about regulatory dynamics are not expressed directly. Take a 3-node network as an example, this network/system has $2^3 = 8$ distinct states, e.g., 000, 001, ..., 111, the Markov chain focuses on estimating the transition probability matrix between any two states, e.g., from state 000 to state 001, but does not provide an explicit interpretation of which variable led the third node to be turned ON. In sum, an advantage of the Boolean network method is it provides an explicit explanation of the dynamics between variables and help researchers understand the psychological dynamics.

The Boolean network method describes the nonlinear temporal dynamics with Boolean expressions and provides direct information about regulatory dynamics, providing a complementary method to the Ising model and the Markov chain model. Depending on the need of how to describe the temporal dynamics, researchers can choose the method that suits their purpose of analysis. To our knowledge, there is not yet a literature that compares the estimation performance of Ising models and Boolean network models, or between Markov chain models and Boolean network models. This is, in part, because the models are based on different theoretical propositions about how the biological, mechanical, or other substrates of the system operate (e.g., using off-on gates or probabilistic gates). Thus, direct comparison of estimated dynamics between these methods is difficult. In the Ising model and Markov chain models the dynamics are represented using continuous scale parameters while in the Boolean network the dynamics are represented using Boolean logic. It is not yet known when the same underlying dynamics might be equally well captured by all three models, or when specific kinds of true models can be identified through comparison of model fits. Simulation studies are needed to identify when each model is most useful theoretically, and when different kinds of true models can be identified and compared.

Boolean Network Control of Nonlinear Dynamics

In this paper, we introduced the Boolean network method as a method to control nonlinear dynamics. Specifically, we use the Boolean network method to find the attractors of the nonlinear system and then design a new Boolean function that will move the system from an undesirable attractor to a desirable attractor basin.

The Boolean network control method presented here provides new opportunities to study and understand the complexity of nonlinear dynamical systems (e.g., human behavior), so that we are no longer limited to the use of linear control theory methods. Linear control theory methods focus on stabilizing the dynamics in the system using linear algebra, designing an input/control into the system to place the real part of the eigenvalues of the dynamics matrix to negative values, so that the discrepancy between the system state and the goal state will exponentially decrease to zero (Lewis et al., 2012; Brunton & Kutz, 2017). That is good, but linear control theory essentially assumes there is only one equilibrium point, a situation that does not fully capture the complexity of the nonlinear dynamics, which are often marked by multi-stability, limit cycle, and even transitions between different regimes (Tang & Bassett, 2018). Hence, the Boolean network method introduced in this paper accommodates the nonlinear dynamics by accounting for multiple attractors and provides control strategies that can direct the system from each undesirable attractor to a desirable attractor.

We also want to make a clarification about the Boolean network control introduced in this paper. In the network control method we introduced here the desired states are constrained to the dynamical systems' naturally occurring attractors and do not include other possible attractors (Campbell & Albert, 2019). Full control would include the possibility to direct the system to any state in the entire state-space. However, realistic control for nonlinear dynamics of biological, social, or technological systems tends to involve only naturally occurring attractors (Mochizuki, Fiedler, Kurosawa, & Saito, 2013; Zañudo & Albert, 2015). Other Boolean network control methods are available to search for control strategies to find a sequence of 0-1 vectors such that the system is directed to any desired state (Akutsu, Hayashida, & Tamura, 2008), so the method can, in principle, be extended to direct the system to non-naturally occurring attractors if there is such a need.

It is worth noting that another approach, fuzzy logic control (FLC), also uses if-then rules to control nonlinear dynamical systems (Yager & Zadeh, 1992), e.g., if anger is ON, then distraction should be ON. FLC is traditionally model-free, which means it does not model the temporal dynamics based on empirical data; instead, it requires human experts to create a set of rules to control the system. Some later development began to extend fuzzy logic control into model-based controllers (Palm, Driankov, & Hallendorn, 1997). Further comparison of FLC and the Boolean network method, as a future step, would be informative for researchers to choose the appropriate control method.

In sum, the Boolean network method can inform the design of control strategies for nonlinear psychological systems. When compared with linear control methods, the Boolean network method can better accommodate the characteristics of nonlinear systems.

Using Network Control Strategy to Inform the Design of Personalized Intervention

In this paper, we proposed the Boolean network method, specially the network control strategies, as a potential model for informing the design of personalized intervention. Using our empirical example, we defined three main patterns of control strategies during a laboratory waiting task based on the availability of control strategy for each network, which can inform the design of personalized behavior modification to achieve developmental goals. The *under-*

developed pattern was distinguished by the finding that, for these networks, there was no control strategy available. Children showing this pattern were stuck in anger, and the bidding and distraction strategies did not help them stop displaying anger. These children may not yet have the capacity for strategies to effectively alter their anger. If this model were applied to intervention, then for this pattern caregivers might be encouraged to foster children's cognitive development and give children practice using strategies. Not all children who are 36 months of age may have the internal resources to regulate anger (Cole, Bendezú, Chow, & Ram, 2017).

In contrast, the *developing pattern* was distinguished by the finding that, for these networks, there were control strategies available. Here, the control method could identify how the enactment of particular strategies might prevent the child from getting stuck in anger. If this model were applied to intervention, caregivers could be reassured that these very young children have developed strategies and they could consider reinforcing their use, e.g., "when I feel angry, I should find a toy to play with." The internalization of such a rule into the child's repertoire of strategies might contribute to further development of effective emotion regulation. Importantly, between-person or between-network comparison can be made by calculating network metrics that quantify emotion regulation characteristics. For example, the number of timesteps needed to achieve the goal of control strategy in the state transition graph could serve as a network metric to describe regulation difficulty. Further development in the network metric area would be informative to select optimal personalized intervention with larger networks (network with many more nodes), where there is more than one control strategy.

The third pattern, the *well-developed pattern*, was distinguished by networks that did not need control strategies that would direct them into a desirable anger = OFF attractor. Children with this pattern appeared to tolerate waiting. They may have regulated anger effectively with a cognitive strategy that could not be observed (e.g., used cognitive reframing, which entails thinking about a problem situation differently) or not experienced sufficient anger to require any strategy. Applied to interventions with caregivers, it may simply be reassurance that the child appears to have learned to tolerate this short type of waiting. Of course, for all three patterns, advice would be based on multiple observations and not one single, short observation.

In sum, the Boolean network method provided person-specific control strategies. These person-specific control strategies then in turn provide personalized behavior modification advice that may contribute to the development of children's emotion regulation capacity and school readiness.

Limitations and Outlooks

Identification of Boolean functions. The identification of Boolean functions uses the criteria of minimal error. The error is the sum of false positives and false negatives. False positives mean predicting the outcome to be 1 but the observed value is 0, and false negatives mean predicting the outcome to be 0, but the observed value is 1. When the observed data can have unbalanced classes, e.g., fewer observations of anger = 1 than that of anger = 0, the error will bias the identification by weighting the false positive and false negative equally. In other words, because there are fewer positive cases than negative cases (e.g., 15 seconds of angry compared to 465 seconds of not angry), the false negative (predicting the case to be negative when it is actually positive) cases might be subsequently fewer than false positive (predicting the case to be positive when it is actually negative). Therefore, the false negative should be more heavily weighted to balance with the false positive. The original biology literature, where Boolean network method has been applied, did not take this into account, possibly because

unbalanced cases are not an issue in biology data. Further research can implement the weighted error to identify the Boolean functions.

The choice of the maximal number of input variables, denoted as $maxK$ in this paper, is another limitation. Preselecting $maxK$ is based on the time complexity of computation if $maxK$ is not a constant, which is a nondeterministic polynomial hard (NP-hard) problem (Akutsu, et al. 2000). The choice of $maxK = 2$ specifically is because when $K = 3$, the system will likely be chaotic, meaning attractors would not be periodic like a limit cycle. This $K = 2$ case is also called the edge of chaos (Derrida & Pomeau, 1986). Preselecting $maxK = 2$ might limit the number of behaviors that can affect emotion by only identifying 1 or 2 behavior that can affect emotion, but it has the benefit of extracting attractors that can inform network control system design and intervention. Hence, it is a tolerable compromise of the accuracy of identifying system dynamics and practical purposes. When theories require use of a larger K , network scientists suggested using a structure-based search algorithm to reduce the computation complexity (Liu & Barabasi, 2016; Zañudo, Yang, Albert, 2017). The structure-based control design relies on the identification of specific network structures – particularly stable motifs wherein structures in the network are not affected by external inputs once they are activated (e.g., anger at $t+1$ predicted by itself at t). This structure-based control design will be a valuable future direction to take to enhance the applicability of the Boolean network method on larger networks.

Probabilistic Boolean Network. An important limitation of the Boolean network method introduced in this paper is the assumption that the dynamics of the system are fully deterministic. This assumption is useful to simplify the modeling process but is not always the most realistic or accurate assumption. One way to incorporate the randomness in the Boolean network is probabilistic Boolean network method (Shmulevich & Dougherty, 2010), which assigns an equal probability for each Boolean function that has minimal error (also available in the BoolNet R package; Müssel, Hopfensitz, & Kestler, 2010). Another way to incorporate stochasticity is to incorporate the process noise in Boolean functions, and this has not been developed to the authors' knowledge. Developing the probabilistic Boolean network method will be a future direction to make the Boolean network method more realistically represent its empirical applications.

Discrete-time Dynamical System Modeling. The Boolean network method is a discrete-time dynamical system modeling, so inherently, it has the flaws of discrete-time modeling (Ryan, Kuiper, & Hamaker, 2019). For example, when the true time-scale of psychological processes is unclear (e.g., how fast can children regulate anger), we usually assume the time-scale is equal with the time-scale measurement, 1 second in the empirical example in this paper, and this assumption dictates how the discrete-time modeling discover temporal dynamics between emotion and behavior. The problem of this assumption is if the true time-scale is slower (e.g., 5 seconds), then we will miss the temporal dynamics between behavior and emotion. This problem can be alleviated by allowing the state transition to be updated asynchronously, meaning not strictly from t to $t+1$ (Albert & Robeva, 2015). This will allow for attractors of multiple time-scales to be discovered. It is worth noting we are not modeling the temporal dynamics with multi-scales though but allowing the time-scale to be changed from 1s to 2s, 3s, by updating the states asynchronously.

Empirical Example. The empirical example in this paper is from an early childhood emotion regulation study. The participants in this sample are somewhat homogeneous in ethnicity. Before generalizing to a larger population and contributing to child emotional development theories, it is useful to engage with other populations, including children with other

ethnicity and cultural background. In terms of variables, we selected anger, bidding, and distraction as the minimal set of variables to illustrate a new methodological approach for studying emotion regulation dynamics. Future work should expand the set of variables to include other emotion and behavior variables (e.g., sad, self-soothing) and/or more granular coding that captures more nuanced expression of emotions or the behaviors that might regulate those emotions (e.g., calm bids, angry bids). For example, our binarization of the anger ratings ignored information about the intensity of the emotion. Surely, the dynamics (i.e., Boolean functions) governing low-intensity anger may be different than the dynamics governing high-intensity anger in that different strategies might be needed to effectively regulate different levels of anger. We consider the heterogeneity in the Boolean networks as a reflection of individual differences in the dynamics of anger regulation; this within-person dynamics may improve the conceptualization of individual differences in emotion regulation, in conjunction with other perspectives, including temperament/personality, parenting, and home environment. Dynamical system models, including the Boolean network method, usually requires long time-series to allow for a good fit; and we were fortunate to have an expensive to produce (second-by-second coding of video) time-series data with 480 observations per child. For time-series data that are shorter, a comprehensive simulation study is needed to obtain guidelines of the minimal number of observations, with respect to the complexity of the network model. For time-series data that are longer (e.g., mobile sensing), a valuable future direction would be to develop this method into a predictive forecasting method using cross-validation and apply it in a wide variety of intensive longitudinal data settings. Furthermore, there could be longitudinal changes in emotion regulation dynamics, often included in the analysis of measurement-burst design, the Boolean network to be further developed into describing patterns in the age-related changes and the development of emotion regulation skills.

Conclusion

We introduced the Boolean network method as a method to model and control multivariate nonlinear dynamical systems in psychology. We synthesized the Boolean network method from the system biology literature into a three-step implementation that includes inference of Boolean functions and construction of Boolean network, extraction of attractors, and design of network control. We then illustrated how the method is applied using empirical data from a study of children's emotion regulation and demonstrate the utility of the Boolean network method for informing the design of advice that might be given to children in their quest to develop better emotion regulation. To our knowledge, this is the first application of Boolean network in psychology literature. Our hope is that the detailed introduction of the methodological steps and empirical demonstration will open up discussions and invite more empirical studies of network control system design for psychological systems.

References

- Akutsu, T., Hayashida, M., & Tamura, T. (2008). Algorithms for Inference, analysis and control of Boolean networks. In K. Horimoto, G. Regensburger, M., Rosenkranz, & H. Yoshida (Eds.), *Algebraic Biology (pp1-15)*. Berlin, Germany: Springer-Verlag.
- Akutsu, T., Miyano, S., & Kuhara, S. (2000). Algorithms for identifying Boolean networks and related biological networks based on matrix multiplication and fingerprint function. *Journal of Computational Biology*, 7(3), 331-343. <https://doi.org/10.1145/332306.332317>.
- Albert, R., & Robeva, R. (2015). Signaling networks: Asynchronous Boolean models. In R. Robeva (Ed.), *Algebraic and Discrete Mathematical Methods for Modern Biology* (pp. 65-91).
- Aracena, J., Gomez, L., & Salinas, L. (2013). Limit cycles and update diagrams in Boolean networks. *Discrete Applied Mathematics*, 161 (1-2), 1-12. <https://doi.org/10.1016/j.dam.2012.07.003>.
- Arnold, M. B. (1960). *Emotion and Personality*. New York, NY: Columbia University Press.
- Benson, L., Ram, N., & Stifter, C.A. (2018). Using fishery models to examine self- and co-regulation processes across multiple time-scales. *Structural Equation Modeling: A Multidisciplinary Journal*, 15(6), 906-923. <https://doi.org/10.1080/10705511.2018.1491313>.
- Berestovksy, N., Nakhleh, L. (2013). An evaluation of methods for inferring Boolean networks from time-series data. *PLoS ONE* 8(6): e66031. <https://doi.org/10.1371/journal.pone.0066031>.
- Boker, S., & Graham, J. (1998). A dynamical systems analysis of adolescent substance use. *Multivariate Behavioral Research*, 33(4), 479-507.
- Brunton, S.L., & Kutz, J.N. (2017). *Data-Driven Science and Engineering: Machine Learning, Dynamical Systems, and Control*. Cambridge, England: Cambridge University Press.
- Campbell, C., & Albert, R. (2019). Edgetic perturbations to eliminate fixed-point attractors in Boolean regulatory networks, *Chaos*, 29(2), 023130. <https://doi.org/10.1063/1.5083060>
- Carver, C.S., & Scheier, M.F. (1998). *On the Self-Regulation of Behavior*. Cambridge, England: Cambridge University Press.
- Carver, C.S., & Scheier, M.F. (2002). Control processes and self-organization as complementary principles underlying behavior. *Personality and Social Psychology Review*, 6(4), 304-315. https://doi.org/10.1207/s15327957pspr0604_05.
- Cheng, D., & Qi, H (2009). Controllability and observability of Boolean control networks. *Automatica*, 40(3), 439–447. https://doi.org/10.1007/978-0-85729-097-7_9.
- Cole, P. M., BendeZú, J. J., Ram, N., & Chow, S.-M. (2017). Dynamical systems modeling of early childhood self-regulation. *Emotion*, 17(4), 684–699. <https://doi.org/10.1037/emo0000268>.
- Cole, P. M., Martin, S. E., & Dennis, T. A. (2004). Emotion regulation as a scientific construct: Methodological challenges and directions for child development research. *Child Development*, 75(2), 317-333. <https://doi.org/10.1111/j.1467-8624.2004.00673.x>.
- Cole, P.M., Ram, N., & English, M.S. (2018). Toward a unifying model of self-regulation: A developmental approach. *The Child Development Perspective*, 13(2), 91-96. <https://doi.org/10.1111/cdep.12316>.

- Cole, P.M., Tan, P.Z., Hall, S.R., Zhang, Y., Crnic, K.A., Blair, C.B., & Li, R. (2011). Developmental changes in anger expression and attention focus: Learning to wait. *Developmental Psychology*, 47(4), 1078-1089. <https://doi.org/10.1037/a0023813>.
- Cole, P.M., Zahn-Waxler, C., & Smith, D.K. (1994). Expressive control during a disappointment: Variations related to preschoolers' behavior problems. *Developmental Psychology*, 30, 835-846. <https://doi.org/10.1037/0012-1649.30.6.835>.
- Cramer, A. O. J., van Borkulo C. D., Giltay E. J., van der Maas H. L. J., Kendler K. S., Scheffer M., & Borsboom, D.. (2016). Major depression as a complex dynamic system. *PLoS ONE* 11(12): e0167490. <https://doi.org/10.1371/journal.pone.0167490>.
- de Hann-Rietdijk, S., Kuppens, P., Bergeman, C.S., Sheeber, L.B., Allen, N.B., & Hamaker, E.L., (2017). On the use of mixed Markov models for intensive longitudinal data. *Multivariate Behavioral Research*, 52(6), 747-767.
- Derrida, B., & Pomeau, Y. (1986). Random networks of automata: A simple annealed approximation. *Europhysics Letters (EPL)*, 1(2), 45-49. <https://doi.org/10.1209/0295-5075/1/2/001>.
- Diaz, A., & Eisenberg, N. (2015). The process of emotion regulation is different from individual differences in emotion regulation: Conceptual arguments and a focus on individual differences. *Psychology Inquiry: An International Journal of the Advancement of Psychological Theory*, 26(1), 37-47.
- Epskamp, S., Maris, G.K.J., Waldorp, L.J., & Borsboom, D. (2018). Network Psychometrics. In P. Irwing, D. Hughes, & T. Booth (Eds.), *The Wiley Handbook of Psychometric Testing, 2 Volume Set: A Multidisciplinary Reference on Survey, Scale and Test Development*. New York: Wiley. <https://doi-org.stanford.idm.oclc.org/10.1002/9781118489772.ch30>.
- Gilliom, M., Shaw, D.S., Beck, J.E., Schonberg, M.A., & Lukon, J.L. (2002). Anger regulation in disadvantaged preschool boys: Strategies, antecedents, and the development of self-control. *Developmental Psychology*, 38(2), 222-235. <https://doi.org/10.1037/0012-1649.38.2.222>.
- Hamming, R. W. (1950). Error detecting and error correcting codes. *The Bell System Technical Journal*, 29 (2), 147-160.
- Henry, T. R., Robinaugh, D., & Fried, E. I. (2020). On the control of psychological networks. <https://doi.org/10.31234/osf.io/7vpz2>
- Ising, E. (1925). Beitrag zur theorie des ferromagnetismus. *Zeitschrift für Physik A Hadrons and Nuclei*, 31, 253-258.
- Kauffman, S. (1969). Metabolic stability and epigenesis in randomly constructed genetic nets. *Journal of Theoretical Biology*, 22, 437-467. [https://doi.org/10.1016/0022-5193\(69\)90015-0](https://doi.org/10.1016/0022-5193(69)90015-0).
- Kauffman, S. (1993). *The Origins of Order: Self-organization and Selection in Evolution*. Oxford, UK: Oxford University Press.
- Kopp, C.B. (1989). Regulation of distress and negative emotions: A developmental view. *Developmental Psychology*, 25, 343-354. <https://doi.org/10.1037/0012-1649.25.3.343>.
- Lähdesmäki, H., Shmulevich, I., & Yli-Harja, O. (2003). On learning gene regulatory networks under the Boolean network model. *Machine Learning*, 52(1), 147-167.
- Lewis, F.L., Vrabie, D.L., & Syrmos, V.L. (2012). *Optimal Control*. Hoboken, NJ: John Wiley & Sons.
- Liu, Y.Y., & Barabasi, A.L. (2016). Control principles of complex networks. *Reviews of Modern Physics*, 88 (3), 3-58.

- Lu, J., Pan, J., Zhang, Q., Dube, L., & Ip, E.H. (2015). Reciprocal Markov modeling of feedback mechanisms between motion and dietary choice using experience-sampling data. *Multivariate Behavioral Research*, 50(6), 584-599. <https://doi.org/10.1080/00273171.2015.1033510>.
- Meyn, S. P., & Tweedie, R.L. (2005). *Markov Chains and Stochastic Stability*. Cambridge, UK: Cambridge University Press.
- Mischel, W. Shoda, Y., & Rodriguez, M.L. (1989). Delay of gratification in children. *Science*, 244(4907), 933-938.
- Mochizuki, A., Fiedler, B., Kurosawa, G., & Saito, D. (2013). Dynamics and control at feedback vertex sets, II: A faithful monitor to determine the diversity of molecular activities in regulatory networks. *Journal of Theoretical Biology*, 335, 130-146. <https://doi.org/10.1016/j.jtbi.2013.06.009>.
- Molenaar, P.C.M. (1987). Dynamic assessment and adaptive optimization of the psychotherapeutic process. *Behavioral Assessment*, 9, 389-416.
- Molenaar, P.C.M., & Nesselroade, J.R. (2015). Systems methods for developmental research. In R.M. Lerner (Ed.), *Handbook of Child Psychology and Developmental Science* (pp 1-31). New York, NY: Wiley.
- Molenaar, P.C.M., & Ram, N. (2010). Dynamic modeling and optimal control of intraindividual variation: A computational paradigm for nonergodic psychological processes. In S.M. Chow, E. Ferrer, & F. Hsieh (Eds.), *Statistical Methods for Modeling Human Dynamics* (pp. 13-37). New York, NY: Routledge.
- Morales, S., Ram, N., Buss, K. A., Cole, P. M., Helm, J. L., & Chow, S. M. (2018). Age-related changes in the dynamics of fear-related regulation in early childhood. *Developmental Science*, 21(5), e12633. <https://doi.org/10.1111/desc.12633>.
- Müssel, C. Hopfensitz, M., & Kestler, H. (2010). BoolNet – An R package for generation, reconstruction, and analysis of Boolean networks. *Bioinformatics*, 26(10), 1378-1380. <https://doi.org/10.1093/bioinformatics/btq124>.
- Palm, R., Driankov, D., & Hellendoorn, H. (1997). *Model Based Fuzzy Control*. Berlin: Springer-Verlag.
- Ramsook, K., A., Benson, L., Ram, N., & Cole, P.M. (2020). Age-related changes in the relation between preschoolers' anger and persistence. *International Journal of Behavioral Development*, 44(3), 216-225. <https://doi.org/10.1177/0165025419866914>.
- Ryan, O., Kuiper, R. M., & Hamaker, E. L. (2019). A continuous-time approach to intensive longitudinal data: What, why, and how? In K. van Montfort, J. H. L. Oud, & M. C. Voelkle (Eds.), *Continuous Time Modeling in the Behavioral and Related Sciences* (pp. 27-54). Springer, Cham. https://doi.org/10.1007/978-3-319-77219-6_2.
- Shmulevich, I. & Dougherty, E.R. (2010). *Probabilistic Boolean Networks: The Modeling and Control of Gene Regulatory Networks*. Society for Industrial and Applied Mathematics.
- Sinclair, K., & Molenaar, P.C.M. (2008). Notes on Optimal control of psychological processes: A new computational paradigm. In *Bulletin de la Societe des Sciences Medicales Luxembourg* (Vol. 1, pp. 13-33). Luxembourg.
- Tang, E., & Bassett, D.S. (2018). Colloquium: Control of dynamics in brain networks. *Reviews of Modern Physics*, 90(3), 031003. <https://doi.org/10.1103/revmodphys.90.031003>.
- Thelen E., & Smith L.B. (2005). Dynamic systems theories. In R.M., Lerner & W. Damon (Eds.), *Handbook of Child Psychology, Volume 1: Theoretical Models of Human*

- Development* (pp.258-312). New York, NY: Wiley.
<https://doi.org/10.1002/9780470147658.chpsy0106>.
- van Borkulo, C. D., Borsboom, D., Epskamp, S., Blanken, T.F., Boschloo, L., Schoevers, R. A., & Waldorp, L. J. (2014). A new method for constructing networks from binary data. *Scientific Reports*, 4, 5918.
- Vaughn, B.E., Kopp, C.B. & Krakow, J.B. (1984). The emergence and consolidation of self-control from eighteen to thirty months of age: Normative trends and individual differences. *Child Development*, 55(3), 990-1004. <https://doi.org/10.2307/1130151>.
- Wang, R.S., & Albert, R. (2011). Elementary signaling modes predict the essentiality of signal transduction network components. *BMC Systems Biology*, 5, Article number: 44. <https://doi.org/10.1186/1752-0509-5-44>.
- Wang, Q., Molenaar, P.C.M., Harsh, S., Freeman, K., Xie, J., Gold, C., Rovine, M., & Ulbrecht, J. (2014). Personalized state-space modeling of glucose dynamics for Type 1 diabetes using continuously monitored glucose, insulin dose, and meal intake. *Journal of Diabetes Science and Technology*, 8(2), 331-345. <https://doi.org/10.1177/1932296814524080>
- Whitesitt, J.E. (2010). *Boolean Algebra and Its Applications*. Mineola, New York: Dover Publications.
- Yager, R.R., & Zadeh, L.A. (1992). *An Introduction to Fuzzy Logic Applications in Intelligent Systems*. New York, NY: Springer.
- Zañudo, J.G.T., & Albert, R. (2015). Cell fate reprogramming by control of intracellular network dynamics. *PLoS Computational Biology*, 11(4), e1004193. <https://doi.org/10.1371/journal.pcbi.1004193>.
- Zañudo, J.G.T., Yang, G., Albert, R., (2017). Structure-based control of complex networks with nonlinear dynamics. *Proceedings of the National Academy of Sciences of the United States of America*, 114(28), 7234-7239.

Table 1 Table of input variable(s) and the outcome of AND ($\wedge$), OR ($\vee$), and NOT rule ($\bar{x}$)

$x(t)$	$y(t)$	$x(t) \wedge y(t)$
0	0	0
0	1	0
1	0	0
1	1	1

$x(t)$	$y(t)$	$x(t) \vee y(t)$
0	0	0
0	1	1
1	0	1
1	1	1

$x(t)$	$\bar{x}(t)$
0	1
1	0

Table 2 Table of state transition

(x_1, x_2)	
t	$t+1$
(0,0)	(0,0)
(0,1)	(0,1)
(1,0)	(1,0)
(1,1)	(0,1)

Table 3 Twenty-two Boolean networks, their associated state transition graph with attractor information, and network control strategies

Index	Boolean Functions			Attractor*	Control Strategy			
	anger	bidding	distraction		Perturb bidding (Strategy, count)	Perturb distraction (Strategy, count)	Not Available (Count)	Not needed (Count)
1	0	bid AND NOT distraction	distraction	A				4
2	0	bid	distraction	B				30
3	0	0	distraction	C				4
4	0	0	0	D				1
5	anger AND NOT bid	bid	distraction	E	Turn bidding ON when anger is ON, 4			
6	anger AND NOT bid	bid	0	F	Turn bidding ON when anger is ON, 1			
7	anger AND NOT bid	0	distraction	G			1	
8	anger AND NOT bid	anger AND bid	distraction	H	Turn bidding ON when anger is ON, 1			
9	anger AND NOT bid	NOT anger AND bid	distraction	I	Turn bidding ON when anger is ON, 1			
10	anger AND NOT distraction	bid	distraction	J		Turn distraction ON when anger is ON, 5		
11	anger AND NOT distraction	bid	NOT bid AND distraction	K		Turn distraction ON when		

						anger is ON,1		
12	anger AND NOT distraction	NOT anger AND bid	distraction	L		Turn distraction ON when anger is ON, 1		
13	anger AND bid	bid	distraction	M	Turn bidding OFF when anger is ON, 9			
14	anger AND distraction	bid AND distraction	distraction	N		Turn distraction OFF when anger is ON, 1		
15	anger AND distraction	bid OR anger	NOT anger AND distraction	O				1
16	anger	bid AND NOT distraction	distraction	P			2	
17	anger	bid	distraction	Q			39	
18	anger	0	distraction	R			2	
19	anger	NOT anger AND bid	distraction	S			4	
20	anger	bid	0	T			3	
21	anger	bid	NOT bid AND distraction	U			1	
22	anger	bid	NOT anger AND distraction	V			1	
Total (117 participants)					16	8	53	40

*Attractors will be represented in the form of state transition graph in Figure 4, labeled with character from A to V in Table 3, with the matching state transition graph in Figure 4 Panel A to Panel V. For example, if column “Attractor” is “C”, then the state transition graph is Figure 4 Panel C.

Figure Captions

Figure 1 Example of a two-node network, consisting of x_1 and x_2 . Panel A is the observed binary time-series of x_1 and x_2 across time. Panel B is the Boolean network and Boolean functions inferred from the binary time-series in Panel A. Panel C is the state transition graph derived from the Boolean functions in Panel B. Panel D is the network control strategy, which is to perturb x_2 when x_1 is ON, and the updated state transition graph from Panel C, where the updated state transition is highlighted by a red arrow. Two-state tuple indicates the state of (x_1, x_2) , e.g., (0,1) means $(x_1=0, x_2=1)$.

Figure 2 Illustration of the design of network control. Panel A is the original Boolean network. Panel B is the associated state transition graph before network perturbation. Panel C is the updated Boolean network using the network control strategy. Panel D is the associated state transition graph if the network control is implemented.

Figure 3 Illustration of a child's model results. Panel A: Binary Time-Series of Emotion System. Binary time-series of anger, bidding, and distraction (colored bars = anger/bidding/distraction = ON, white space = anger/bidding/distraction = OFF). Panel B: Inferred Boolean functions. Inferred Boolean functions based on the time-series in Panel A. Panel C: State Transition Graph and Attractors. Each dot represents a state of the system (a 3-digit string ordered as anger, bidding, and distraction). Six attractors were extracted, which has a self-loop on the dot, e.g., 000 has a self-loop indicating it is an attractor. Different colors indicate different attractors and its basin (if applicable), and the dashed lines with arrows indicate the direction of state transition from t to $t+1$. Panel D: Network Control Strategies. One control strategy is found to modify the behavior of distraction with a new Boolean function. Panel E: The updated state transition graph shows how undesirable attractor is eliminated and the state transition from an undesirable state to a desirable attractor basin, e.g., an undesirable attractor 100 (anger is ON, highlighted in green) will transition to the state 101 and eventually go to attractor 001 (anger is OFF).

Figure 4 Illustration of twenty-two state transition graphs. In each panel, each color indicates a different attractor and its basin, and the self-loop indicates an attractor, and dashed arrows indicate the direction of state transition, e.g., Panel A shows a state transition graph with three attractors (0,1,0) in green, (0,0,1) in red, (0,0,0) in blue, and (0,1,1) transitions to (0,0,1). The variables in the tuple/parenthesis are anger, bid, distraction.

Figure 1

A. Observed time-series

Variables at time t		
	x_1	x_2
$t1$	1	0
$t2$	1	0
$t3$	0	1
$t4$	0	1
$t5$	1	1
$t6$	0	1
...	...	...

B. Boolean network

Boolean functions:

$$x_1(t+1) = x_1(t) \wedge \overline{x_2(t)}$$

$$x_2(t+1) = x_2(t)$$

C. State transition graph

D. Network Control

Perturb x_2 when x_1 is ON

Figure 2

A. Boolean functions:

$$x_1(t+1) = x_1(t) \wedge \overline{x_2(t)}$$

$$x_2(t+1) = x_2(t)$$

B. State transition graph

C. New Boolean functions:

$$x_1(t+1) = x_1(t) \wedge \overline{x_2(t)}$$

$$x_2(t+1) = x_2(t) \vee (x_1(t) \wedge \overline{x_2(t)})$$

D. New State transition graph

Figure 3

A. Binary Time-Series of the Emotion System

B. Inferred Boolean Functions

$$anger(t+1) = anger(t) \wedge \overline{distraction(t)}$$

$$bidding(t+1) = bidding(t)$$

$$distraction(t+1) = distraction(t)$$

D. Network Control Strategies

- Turn distraction ON when anger is ON
- Update Boolean function of distraction to:
 $distraction(t) \vee (anger(t) \wedge \overline{distraction(t)})$

C. State Transition Graph and Attractors

E. Updated state transition graph

Figure 4